NETWORK ANALYSIS

Theory, Example and Practice

GATE GUIDE

NETWORK ANALYSIS

Theory, Example and Practice

R. K. Kanodia

Ashish Murolia

JHUNJHUNUWALA

JAIPUR

Exclusive Sales Counter & BO:
NODIA & COMPANY
55, Suryalok Complex, Gunfoundry, Abids, Hyderabad – 500001.
Phone : 040 – 64582577

Network Analysis : Theory, Example & Practice, 1e
R. K. Kanodia, Ashish Murolia

AA1213

JHUNJHUNUWALA
B-8, Dhanshree Tower Ist, Central Spine, Vidyadhar Nagar, Jaipur – 302023
Ph : +91–141–2101150.
www.nodia.co.in
email : enquiry@nodia.co.in

Printed By: Nodia & Company, Jaipur

The GATE GUIDE

The GATE GUIDE is an exclusive series of books authored by RK Kanodia & Ashish Murolia, published by JHUNJHUNUWALA. GATE GUIDE is intended to provide best content to the students preparing for GATE(Electronics & Communication Engineering). The GATE examination consists of multiple choice problems which are tricky, conceptual and evalutes the fundamental understanding of the subject. As an GATE aspirant your study should be emphasized on the following points all which are incorporated in each GATE GUIDE.

Brief and Explicit Theory which covers all the Topics: The syllabus of GATE examination includes all the subjects of under graduation which you have to study in a short span of your preparation. Therefore, theory should be point-to-point and explicit which develops the fundamentals of the subject. Additionally, it should give you the whole coverage of the syllabus.

Concepts & Formulas: The questions appeared in GATE are numerical as well as conceptual. Your study must incorporates all the concepts and formulas which should be highlighted for a quick reading.

Step-by-step Problem Solving Methodology: For solving different kind of numerical problems, a particular methodology should be followed. Note that for a specific problem alternate methods can be used. The best method is one which is much simpler and less time consuming.

Well-explained Examples: Solved examples gives a good understanding of the solution methodologies. They enhance the problem solving skills. Also, it makes you to choose the best solution between alternate methods.

Practice Exercise: Only theory is not sufficient for a good score. You need to practice as much questions as you can. Remember that, similar questions do not give the whole breadth of the syllabus. There should be a variety of questions which covers all the topics.

GATE GUIDE is first of its kind ever published for GATE aspirants. GATE GUIDE is available in the following subjects:

★ Signals & System
★ Network Analysis
★ Communication System
★ Control System

Preface To The First Edition

The stupendous response for the first title of the series GATE GUIDE Signals & Systems encouraged us to present GATE GUIDE Network Analysis. Over the last few decades, numerous text books have been published on this subject. But, still the students face difficulties when they begin preparing for an competitive examination like GATE. The reason behind is that most texts are too large, tedious and based on universities syllabus. There is no book which has been designed thoroughly for an engineering competitive examination.

This book is intended to provide a complete and straight forward coverage of the subject of Network Analysis for the GATE examination. The book has been categorized into fifteen chapters which covers whole breadth of the syllabus of Network Analysis for GATE Electronics & Communication Engineering. The notable feature of this book is the explicit theory, problem solving methodologies, solved examples and practice exercises given in each chapter. The text is written in very clear and matter-of-fact style. We try to prevent excessive text which hides core concepts of the subject and dissipates a lot of time. Important formulas and concepts are highlighted in the text screen for quick reading.

Problem solving is fundamental to the study of circuit analysis. Each chapter contains step-by-step problem solving methodology, by following which student feels enable to solve almost each variety of problem in circuit analysis. Solved examples are incorporated after each methodology and are solved by using same procedure given in methodology. Solved examples strengthen your problem solving skills and makes you confident in solving problems. We have taken the solved examples in form of multiple choices questions considering the fact that GATE examination is based on multiple choice questions only.

Once you go through text and solved examples the practice exercises become easier to solve. Practice exercises are graded as *exercise A* and *exercise B* on the basis of complexity of questions. Also, exercise A organizes question in particular order of theory, whereas in exercise B questions are ordered randomly. Answer key of each practice exercise is given at the end of the book to enlighten you, if you get a correct answer for the problem. Each question of practice exercise is complied as exam like as possible.

Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge the comments, criticism and suggestion from the users of this book which leads to some improvement. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com

Wish you all the success in conquering GATE.

Authors

AT A GLANCE

1. Brief Theory

Each chapter comprises brief theory covering all the topics. It is very explicit and provides a clear understanding of the topics.

2. Problem Solving Method

a step by step approach for problem solving procedures.

3. Solved Example (Multiple Choice)

Each topic is followed by a Multiple choice solved example which has a significant relevance with theory.

Initial current through inductor is $i_L(0^-) = 2\,\text{A}$.
Hence (C) is correct option.

14.6 CIRCUIT ANALYSIS IN THE s-DOMAIN

All the circuit analysis techniques that we have studied for pure resistive networks may be used in s-domain analysis. The node voltage method, mesh current method, source transformations, and Thevenin-Norton equivalents are all valid techniques in the s-domain. These can be applied using same methodologies as we discussed for resistive networks.

The step-by-step procedure of circuit analysis in the s-domain is given below.

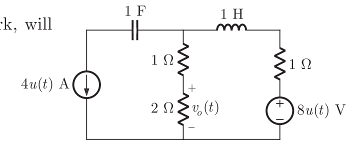
METHODOLOGY

1. Draw the circuit into s-domain by substituting an s-domain equivalent for each circuit element. The inductors and capacitors are replaced by their equivalent discussed in previous section.
2. Apply any circuit analysis technique to obtain the desired voltage or current in the s-domain.
3. Take inverse Laplace transform to convert the voltages and/or currents back to the time domain.

For transient circuits, first we find initial capacitor voltages and inductor currents. To obtain this, we draw the circuit for $t < 0$, by replacing all capacitors with open circuits and all inductors with short circuits. After that follow the steps 1, 2 and 3.

EXAMPLE

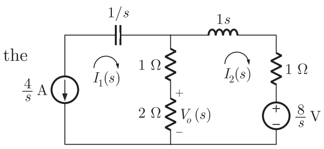
For $t > 0$, the voltage $v_o(t)$ in the following network, will be



(A) $v_o(t) = 2(1 - e^{-4t})\,u(t)\,\text{V}$
(B) $v_o(t) = (-8 + 10e^{-4t})\,u(t)\,\text{V}$
(C) $v_o(t) = (8 - 10e^{-4t})\,u(t)\,\text{V}$
(D) $v_o(t) = 2(1 - 5e^{-4t})\,u(t)\,\text{V}$

SOLUTION :

Step 1: Taking zero initial condition and transforming the circuit into s-domain as shown in figure.



Step 2: Applying mesh analysis to the circuit

Mesh 1: $I_1(s) = -4/s$

4. Marginal Notes

Marginal notes are extra source of learning. They emphasize useful concepts, summarized text and indicates common mistakes that students need to avoid.

5. Text Screen

Useful concepts, theorems and formulas are highlighted into text screen for a quick reading.

For any linear resistive circuit, any output voltage or current, denoted by the variable y, is related linearly to the independent sources(inputs), i.e.,

$$y = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$

where $x_1, x_2 .... x_n$ are the voltage and current values of the independent sources in the circuit and a_1 through a_m are properly dimensioned constants.

Thus, a linear circuit is one whose output is linearly related (or directly proportional) to its input. For example, consider the linear circuit shown in figure 5.2.1. It is excited by an input voltage source V_s, and the current through load R is taken as output(response).

Suppose $V_s = 5\text{ V}$ gives $I = 1\text{ A}$. According to the linearity principle, $V_s = 10\text{ V}$ will give $I = 2\text{ A}$. Similarly, $I = 4\text{ mA}$ must be due to $V_s = 20\text{ mV}$. Note that ratio V_s/I remains constant, since the system is linear.

We know that the relationship between power and voltage (or current) is not linear. Therefore, linearity does not applicable to power calculations..

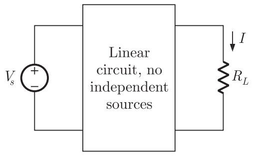
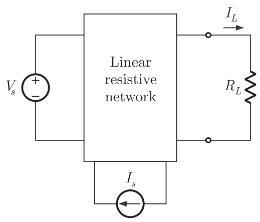


Fig 5.2.1 A linear circuit

► EXAMPLE

For the circuit shown in figure, some measurements are made and listed in the table below.



	V_s	I_s	I_L
1.	7 V	3 A	1 A
2.	9 V	1 A	3 A

If $V_s = 10\text{ V}$ and $I_s = 4\text{ V}$, then the value of I_L will be

(A) 1.6 A

(B) 4.4 A

(C) 3.2 A

(D) 6.4 A

6. Practice Exercise

Practice exercises covers variety of problems from each topic that enhance your confidence level. Practice exercises are divided into two levels on the basis of complexity.

PRACTICE A

MCQ 8.1.1 The current in an RLC circuit is described by the equation

$$\frac{d^2 i(t)}{dt^2} + 8\frac{di(t)}{dt} + 16 i(t) = 0$$

What is the natural frequency for the circuit ?

(A) -2 rad/sec (B) -8 rad/sec
(C) -4 rad/sec (D) -12 rad/sec

MCQ 8.1.2 The current in a series RLC network is given by

$$i(t) = Ae^{-5t} + Be^{-t}, \quad A \ \& \ B \text{ are constants}$$

The damping factor will be equal to

(A) 2.68 (B) 0.6
(C) 1.2 (D) 1.34

MCQ 8.1.3 A series RLC circuit has $R = 4\,\Omega$ and $C = 2$ F. The value of L so that the circuit is critically damped, will be

(A) 4 H (B) 2 H
(C) 16 H (D) 8 H

Statement for Questions 4-6 :

A parallel RLC circuit has the following parameters; $R = 1\,\text{k}\Omega$, $L = 12.5$ H, and $C = 2\,\mu$F.

MCQ 8.1.4 What type of damping does the circuit exhibit ?

(A) critical damping (B) under damping
(C) over damping (D) none of these

MCQ 8.1.5 What value of R will cause a damped frequency of 120 rad/sec ?

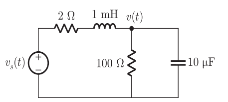
(A) 3.12 kΩ (B) 1.56 kΩ
(C) 6.41 kΩ (D) 12.82 kΩ

MCQ 8.1.6 What value of R will result in a critically damped response ?

(A) 2 kΩ (B) 2.5 kΩ
(C) 5 kΩ (D) 1.25 kΩ

PRACTICE B

MCQ 8.2.1 The differential equation for the circuit shown below is



(A) $v''(t) + 3000v'(t) + 1.02 \times 10^8 v(t) = 10^8 v_s(t)$
(B) $v''(t) + 1000v'(t) + 1.02 \times 10^8 v(t) = 10^8 v_s(t)$
(C) $\frac{v''(t)}{10^8} + \frac{2v'(t)}{10^5} + 1.02 v(t) = v_s(t)$
(D) $\frac{v''(t)}{10^8} + \frac{2v'(t)}{10^5} + 1.98 v(t) = v_s(t)$

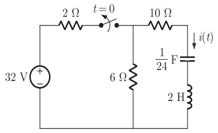
MCQ 8.2.2 In a parallel RLC circuit the voltage across inductor and current through the capacitor are given as

$$v_L(t) = (20e^{-4t}\cos 2t - 80e^{-4t}\sin 2t)\ \text{V}$$
$$i_C(t) = (-6e^{-4t}\cos 2t + 7e^{-4t}\sin 2t)\ \text{A}$$

What are the values of parameters R, L and C ?

(A) $R = 10\,\Omega$, $L = 1$ H, $C = 50$ mF
(B) $R = 5\,\Omega$, $L = 2$ H, $C = 25$ mF
(C) $R = 3.33\,\Omega$, $L = 1.33$ mH, $C = 37.5$ mF
(D) $R = 5\,\Omega$, $L = 1$ H, $C = 50$ mF

MCQ 8.2.3 In the following network, the switch opens instantaneously at $t = 0$. The current $i(t)$ for $t > 0$ will be



SYLLABUS

GATE ELECTRONICS & COMMUNICATION ENGINEERING

Networks:

Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

IES ELECTRONICS & TELECOMMUNICATION ENGINEERING

Networks Theory:

Network analysis techniques; Network theorems, transient response, steady state sinusoidal response; Network graphs and their applications in network analysis; Tellegen’s theorem. Two port networks; Z, Y, h and transmission parameters. Combination of two ports, analysis of common two ports. Network functions : parts of network functions, obtaining a network function from a given part. Transmission criteria : delay and rise time, Elmore’s and other definitions effect of cascading. Elements of network synthesis.

CONTENTS

CHAPTER 10
AC POWER ANALYSIS 481

CHAPTER 11
THREE PHASE CIRCUITS 529

CHAPTER 12
MAGNETICALLY COUPLED CIRCUITS 569

CHAPTER 15

TWO PORT NETWORK 707

CHAPTER 5

CIRCUIT THEOREMS

5.1 INTRODUCTION

In this chapter we study the methods of simplifying the analysis of more complicated circuits. We shall learn some of the circuit theorems which are used to reduce a complex circuit into a simple equivalent circuit. This includes Thevenin theorem and Norton theorem. These theorems are applicable to linear circuits, so we first discuss the concept of circuit linearity.

5.2 LINEARITY

A system is linear if it satisfies the following two properties

Homogeneity Property :

The homogeneity property requires that if the input (excitation) is multiplied by a constant, then the output (response) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input I to the output V,

$$V = IR$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$kIR = kV$$

Additivity Property :

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$V_1 = I_1 R \qquad \text{(Voltage due to current } I_1\text{)}$$

and

$$V_2 = I_2 R \qquad \text{(Voltage due to current } I_2\text{)}$$

then, by applying current $(I_1 + I_2)$ gives

$$V = (I_1 + I_2) R = I_1 R + I_2 R$$

$$= V_1 + V_2$$

These two properties defining a linear system can be combined into a single statement as

For any linear resistive circuit, any output voltage or current, denoted by the variable y, is related linearly to the independent sources(inputs), i.e.,

$$y = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$

where $x_1, x_2 .... x_n$ are the voltage and current values of the independent sources in the circuit and a_1 through a_m are properly dimensioned constants.

We know that the relationship between power and voltage (or current) is not linear. Therefore, linearity does not applicable to power calculations..

Thus, a linear circuit is one whose output is linearly related (or directly proportional) to its input. For example, consider the linear circuit shown in figure 5.2.1. It is excited by an input voltage source V_s, and the current through load R is taken as output(response).

Suppose $V_s = 5\text{ V}$ gives $I = 1\text{ A}$. According to the linearity principle, $V_s = 10\text{ V}$ will give $I = 2\text{ A}$. Similarly, $I = 4\text{ mA}$ must be due to $V_s = 20\text{ mV}$. Note that ratio V_s/I remains constant, since the system is linear.

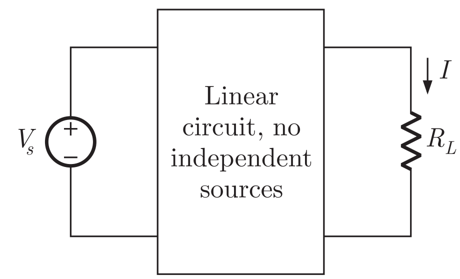
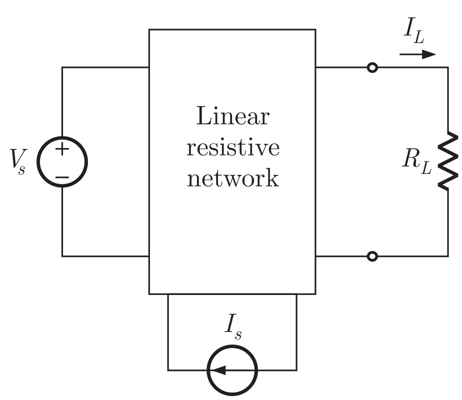


Fig 5.2.1 A linear circuit

▶ EXAMPLE

For the circuit shown in figure, some measurements are made and listed in the table below.



	V_s	I_s	I_L
1.	7 V	3 A	1 A
2.	9 V	1 A	3 A

If $V_s = 10\text{ V}$ and $I_s = 4\text{ V}$, then the value of I_L will be

(A) 1.6 A

(B) 4.4 A

(C) 3.2 A

(D) 6.4 A

SOLUTION :

Circuit is linear, so the linear equation relating inputs V_s and I_s to output I_L is given by

$$I_L = AV_s + BI_s$$

now, using the values of table

$$1 = 7A + 3B \quad ...(i)$$

$$3 = 9A + B \quad ...(ii)$$

Solving equation (i) and (ii)

$$A = 0.4,\ B = -0.6$$

So, $I_L = 0.4V_s - 0.6I_s$

For $V_s = 10\text{ V}$ and $I_s = 4\text{ V}$

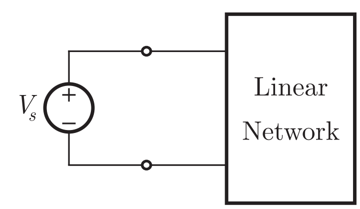
$$I_L = 0.4(10) - 0.6(4)$$

$$= 4 - 2.4 = 1.6\text{ A}$$

Hence (A) is correct option.

▶ EXAMPLE

The linear network in the figure contains resistors and dependent sources only. When $V_s = 10\text{ V}$, the power supplied by the voltage source is 40 W. What will be the power supplied by the source if $V_s = 5\text{ V}$?



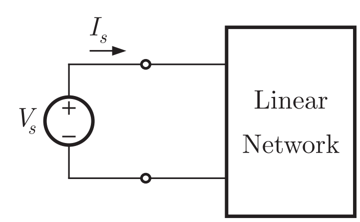
(A) 20 W

(B) 10 W

(C) 40 W

(D) can not be determined

SOLUTION :



For, $V_s = 10\text{ V},\ P = 40\text{ W}$

So, $I_s = \dfrac{P}{V_s} = \dfrac{40}{10} = 4\text{ A}$

Now, $V_s' = 5\text{ V}$, so $I_s' = 2\text{ A}$ (From linearity)

New value of the power supplied by source is

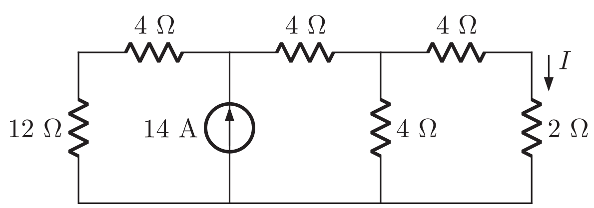
$$P_s' = V_s' I_s' = 5 \times 2 = 10\text{ W}$$

Note: Linearity does not apply to power calculations.

Hence (B) is correct option.

▶ EXAMPLE

In the circuit shown below, the value of current I will be given by



(A) 2.5 A

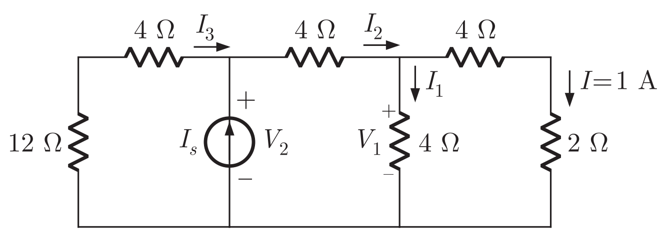
(B) 1.5 A

(C) 4 A

(D) 2 A

SOLUTION :

We solve this problem using linearity and assumption that $I = 1\text{ A}$.



$$V_1 = 4I + 2I = 6\text{ V} \qquad \text{(Using KVL)}$$

$$I_2 = I_1 + I = \frac{V_1}{4} + I = \frac{6}{4} + 1 = 2.5\text{ A} \qquad \text{(Using KCL)}$$

$$V_2 = 4I_2 + V_1 = 4(2.5) + 6 = 16\text{ V} \qquad \text{(Using KVL)}$$

$$I_s + I_3 = I_2 \qquad \text{(Using KCL)}$$

$$I_s - \frac{V_2}{4+12} = I_2$$

$$I_s = \frac{16}{16} + 2.5 = 3.5\text{ A}$$

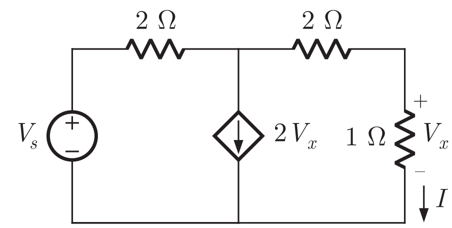
When $I_s = 3.5\text{ A}$, $I = 1\text{ A}$

But $I_s = 14\text{ A}$, so $I = \frac{.1}{3.5} \times 14 = 4\text{ A}$

Hence (C) is correct option.

▶ EXAMPLE

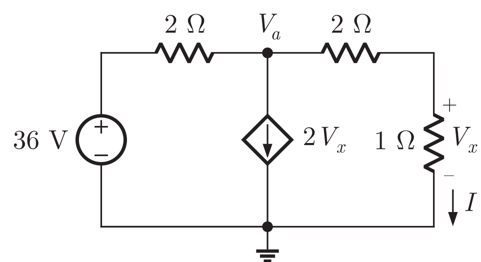
In the circuit, what are the values of I for $V_s = 36$ V and $V_s = 72$ V respectively ?



(A) 36 A, 18 A (B) 18 A, 72 A

(C) 4 A, 8 A (D) 36 A, 9 A

SOLUTION :



Applying nodal analysis at node a

$$\frac{V_a - 36}{2} + 2V_x + \frac{V_a}{(2+1)} = 0$$

$$3V_a - 108 + 12V_x + 2V_a = 0$$

$$5V_a + 12V_x = 108 \qquad \text{...(i)}$$

$$V_x = \frac{1}{2+1}(V_a) \quad \text{(Using voltage division)}$$

$$V_a = 3V_x$$

Substituting $V_a = 3V_x$ into equation (i),

$$5(3V_x) + 12V_x = 108$$

$$27V_x = 108$$

$$V_x = 4 \text{ V}$$

So, current $$I = \frac{V_x}{1} = \frac{4}{1} = 4 \text{ A}$$

Since the circuit is linear, So when $V_s = 72$ V, then

$$I_2 = 4 \times 2 = 8 \text{ A}$$

Hence (C) is correct option.

5.3 SUPERPOSITION

It states that, in any linear circuit containing multiple independent sources the total current through or voltage across an element can be determined by algebraically adding the voltage or current due to each independent source acting alone with all other independent sources set to zero.

The number of circuits required to solve a network. using superposition theorem is equal to the number of independent sources present in the network.

An independent voltage source is set to zero by replacing it with a 0 V source(short circuit) and an independent current source is set to zero by replacing it with 0 A source(an open circuit). The following methodology

Superposition theorem can not be applied to power calculations since power is not a linear quantity.

illustrates the procedure of applying superposition to a given circuit

METHODOLOGY

1. Consider one independent source (either voltage or current) at a time, short circuit all other voltage sources and open circuit all other current sources.
2. Dependent sources can not be set to zero as they are controlled by other circuit parameters.
3. Calculate the current or voltage due to the single source using any method (KCL, KVL, nodal or mesh analysis).
4. Repeat the above steps for each source.
5. Algebraically add the results obtained by each source to get the total response.

EXAMPLE

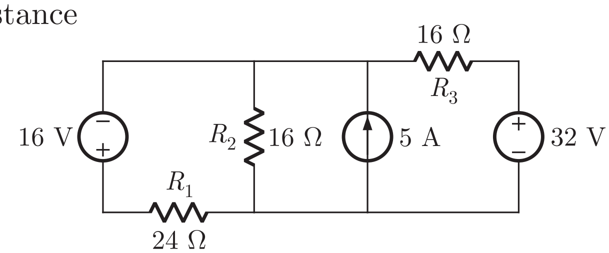
In the circuit of figure, the voltage drop across the resistance R_2 will be equal to

(A) 46 volt

(B) 38 volt

(C) 22 volt

(D) 14 volt

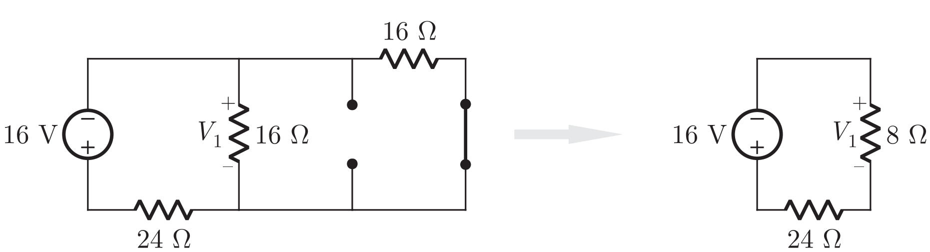


SOLUTION :

The circuit has three independent sources, so we apply superposition theorem to obtain the voltage drop.

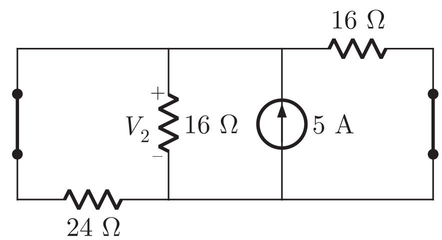
Due to 16 V source only : (Open circuit 5 A source and Short circuit 32 V source)

Let voltage across R_2 due to 16 V source only is V_1.



Using voltage division

$$V_1 = -\frac{8}{24+8}(16) = -4\text{ V}$$



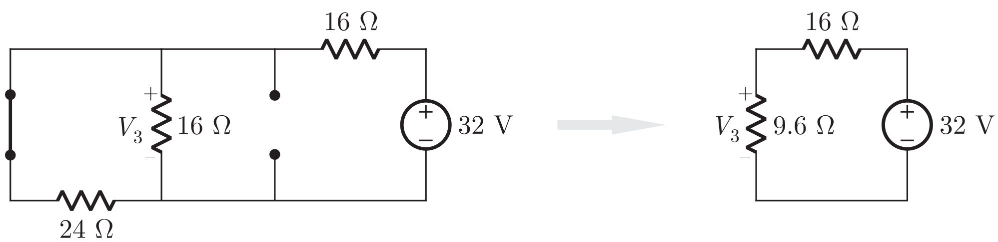
Due to 5 A source only : (Short circuit both the 16 V and 32 V sources)

Let voltage across R_2 due to 5 A source only is V_2.

$$V_2 = (24\,\Omega\,||\,16\,\Omega\,||\,16\,\Omega) \times 5$$
$$= 6 \times 5 = 30\text{ volt}$$

Due to 32 V Source Only : (Short circuit 16 V source and open circuit 5 A source)

Let voltage across R_2 due to 32 V source only is V_3



Using voltage division

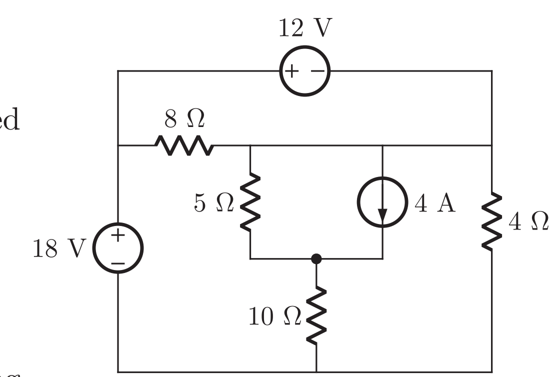
$$V_3 = \frac{9.6}{16+9.6}(32) = 12\text{ V}$$

By superposition, the net voltage across R_2 is

$$V = V_1 + V_2 + V_3$$
$$= -4 + 30 + 12$$
$$= 38\text{ volt}$$

Hence (B) is correct option.

EXAMPLE



In the circuit shown in the given figure, power dissipated in 4 Ω resistor is

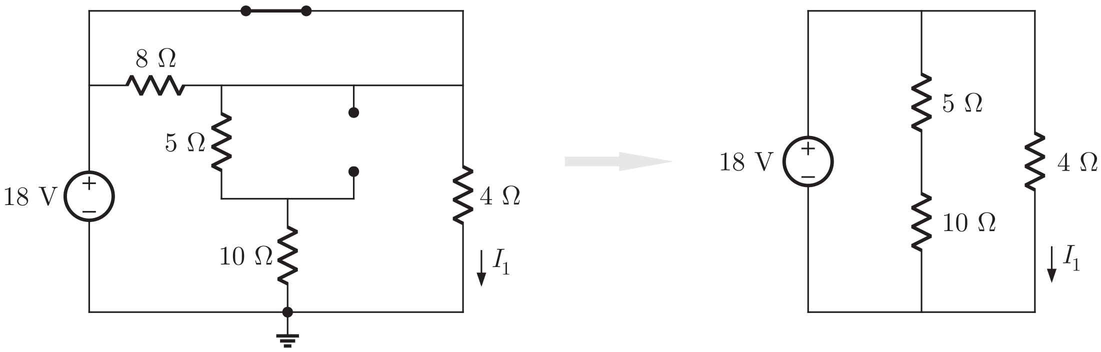
(A) 225 W (B) 121 W

(C) 9 W (D) none of these

SOLUTION :

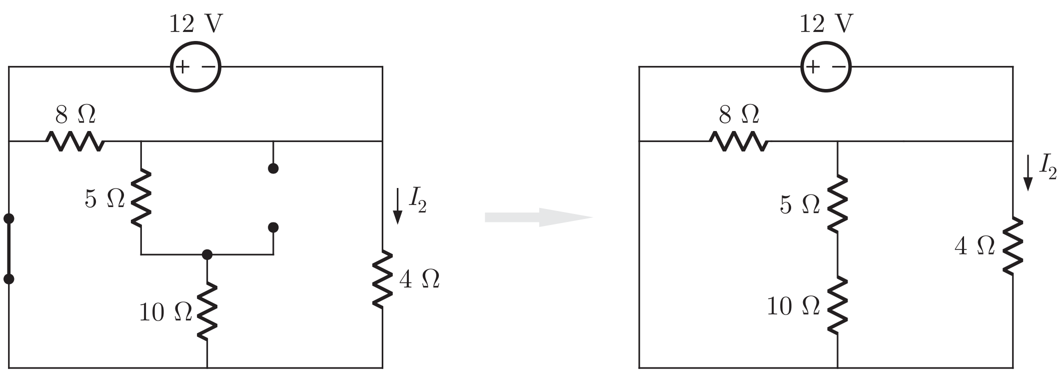
First, we find current I in the 4 Ω resistors using superposition.

Due to 18 V Source Only : (Open circuit 4 A and short circuit 12 V source)



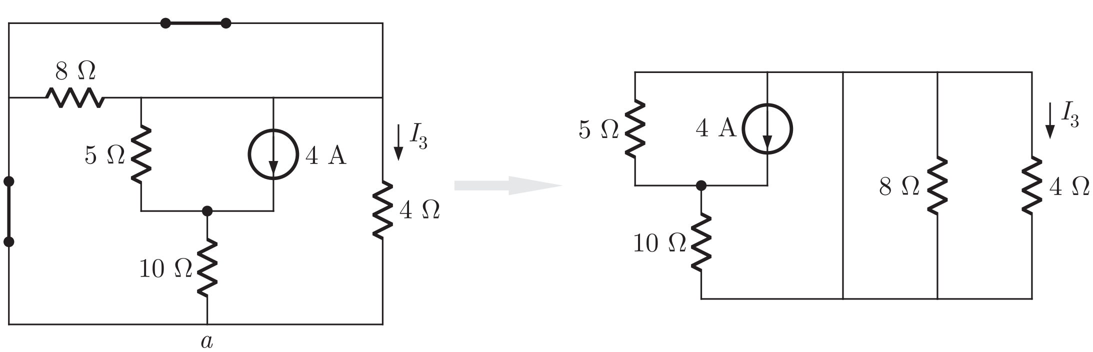
$$I_1 = 18/4 = 4.5\text{ A}$$

Due to 12 V Source Only : (Open circuit 4 A and short circuit 18 V source)



$$I_2 = -12/4 = -3\text{ A}$$

Due to 4 A Source Only : (Short circuit 12 V and 18 V sources)



$$I_3 = 0 \qquad \text{(Due to short circuit)}$$

So, $I = I_1 + I_2 + I_3 = 4.5 - 3 + 0 = 1.5\text{ A}$

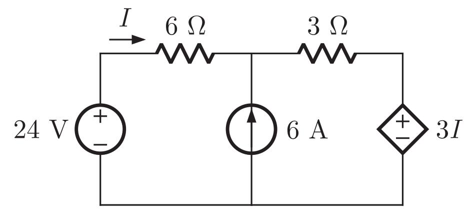
Power dissipated in 4 Ω resistor

$$P_{4\,\Omega} = I^2(4) = (1.5)^2 \times 4 = 9\text{ W}$$

Hence (C) is correct option.

▶ EXAMPLE

For the following circuit, value of current I is given by



(A) 0.5 A

(B) 3.5 A

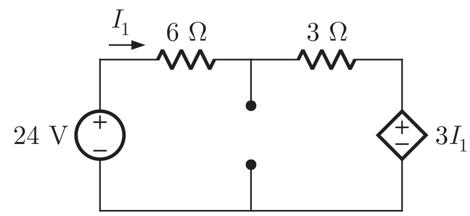
(C) 1 A

(D) 2 A

SOLUTION :

We obtain I using superposition. Note that while applying superposition we do not set dependent source to zero.

Due to 24 V source only: (Open circuit 6 A)

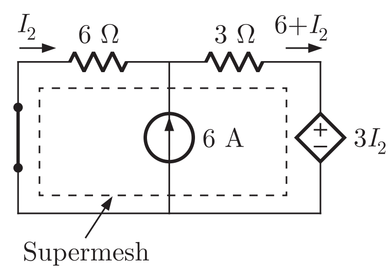


Applying KVL

$$24 - 6I_1 - 3I_1 - 3I_1 = 0$$

$$I_1 = \frac{24}{12} = 2\text{ A}$$

Due to 6 A source only: (Short circuit 24 V source)



Applying KVL to supermesh

$$-6I_2 - 3(6 + I_2) - 3I_2 = 0$$

$$6I_2 + 18 + 3I_2 + 3I_2 = 0$$

$$I_2 = -\frac{18}{12} = -\frac{3}{2}\text{ A}$$

From superposition, $I = I_1 + I_2$

$$= 2 - \frac{3}{2} = \frac{1}{2}\text{ A}$$

Hence (A) is correct option.

5.4 SOURCE TRANSFORMATION

It states that an independent voltage source V_s in series with a resistance R is equivalent to an independent current source $I_s = V_s/R$, in parallel with a resistance R.

or

An independent current source I_s in parallel with a resistance R is equivalent to an independent voltage source $V_s = I_s R$, in series with a resistance R.

Figure 5.4.1 shows the source transformation of an independent source. The following points are to be noted while applying source transformation.

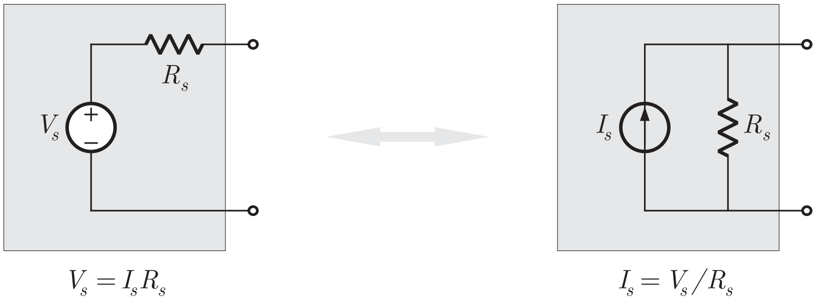


Fig 5.4.1 Source Transformation of independent sources

1. Note that head of the current source arrow corresponds to the +ve terminal of the voltage source. The following figure illustrates this

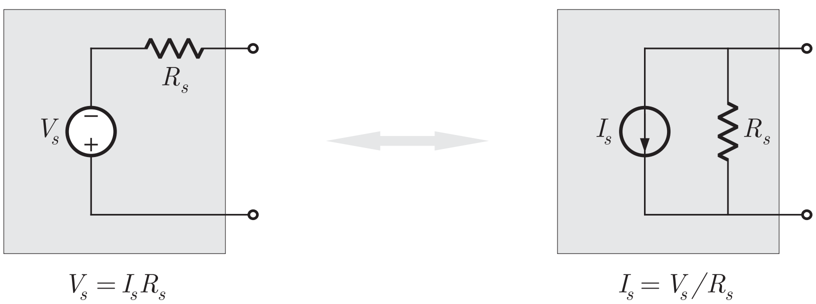
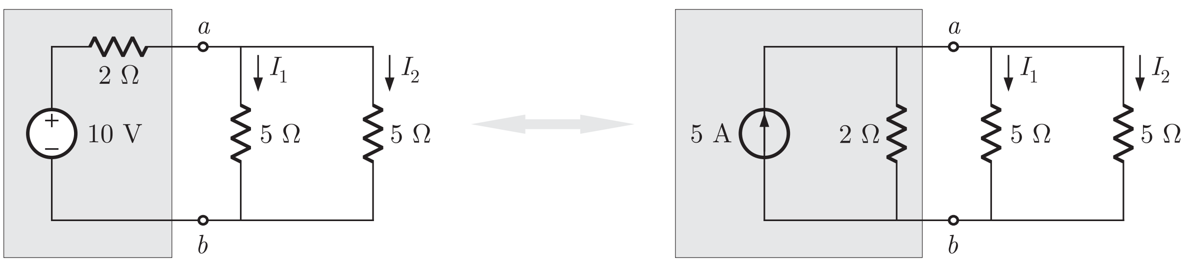


Fig 5.4.2

2. Source conversion are equivalent at their external terminals only i.e. the voltage-current relationship at their external terminals remains same. The two circuits in figure 5.4.3a and 5.4.3b are equivalent, provided they have the same voltage-current relation at terminals *a*-*b*



(a) Circuit with a voltage source

(b) Equivalent circuit when the voltage source is transformed into current source

Fig 5.4.3 An example of source transformation

3. Source transformation is not applicable to ideal

voltage sources as $R_s = 0$ for an ideal voltage source. So, equivalent current source value $I_s = V_s/R \rightarrow \infty$. Similarly it is not applicable to ideal current source because for an ideal current source $R_s = \infty$, so equivalent voltage source value will not be finite.

▶ EXAMPLE

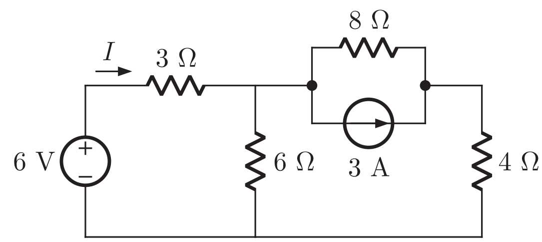
The value of current I in the circuit, is equal to

(A) 2/7 A

(B) 1 A

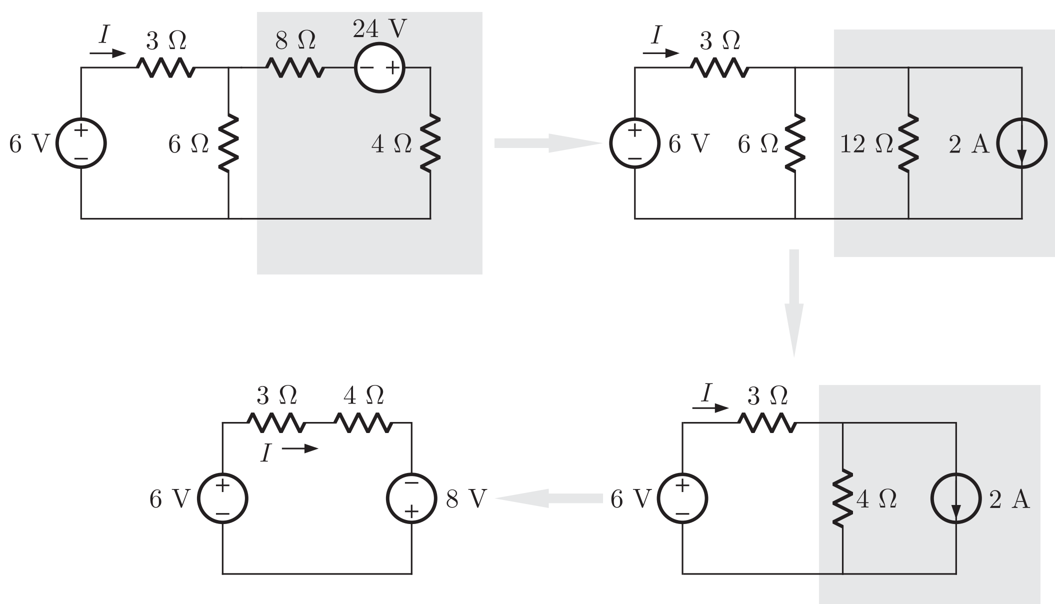
(C) 2 A

(D) 4 A



SOLUTION :

Using source transformation, we can obtain I in following steps.

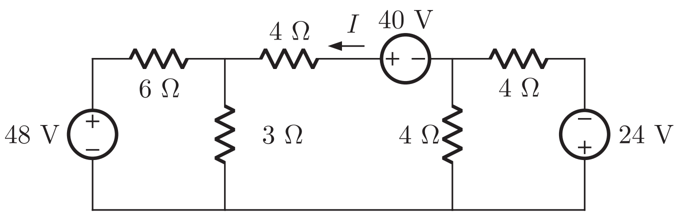


$$I = \frac{6+8}{3+4} = \frac{14}{7} = 2 \text{ A}$$

Hence (C) is correct option.

▶ EXAMPLE

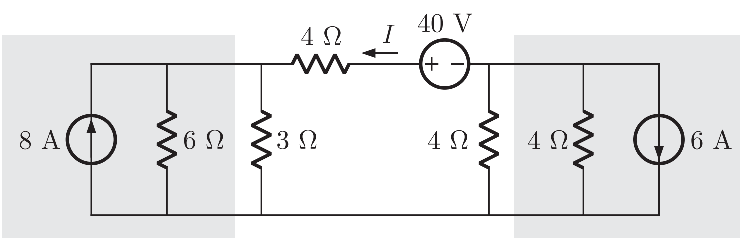
What is the value of current I in the circuit shown below ?



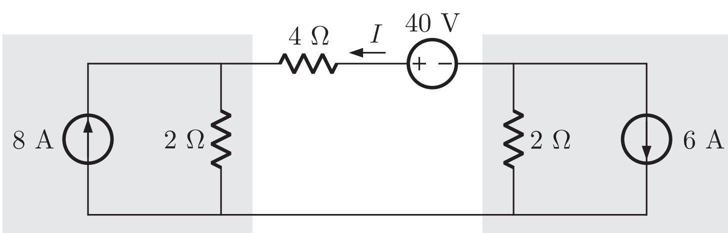
(A) 8.5 A
(B) 4.5 A
(C) 1.5 A
(D) 5.5 A

SOLUTION :

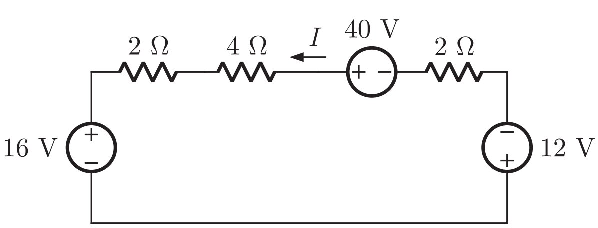
Using source transformation of 48 V source and the 24 V source



using parallel resistances combination



Source transformation of 8 A and 6 A sources



Writing KVL around anticlock wise direction

$$-12 - 2I + 40 - 4I - 2I - 16 = 0$$

$$12 - 8I = 0$$

$$I = \frac{12}{8} = 1.5\text{ A}$$

Hence (C) is correct option.

5.4.1 Source Transformation For Dependent Source

Source transformation is also applicable to dependent source in the same manner as for independent sources.

It states that an dependent voltage source V_x in series with a resistance R is equivalent to a dependent current source $I_x = V_x/R$, in parallel with a resistance R, keeping the controlling voltage or current unaffected.

or,

A dependent current source I_x in parallel with a resistance R is equivalent to an dependent voltage source $V_x = I_x R$, in series with a resistance R, keeping the controlling voltage or current unaffected.

Figure 5.4.4 shows the source transformation of an dependent source.

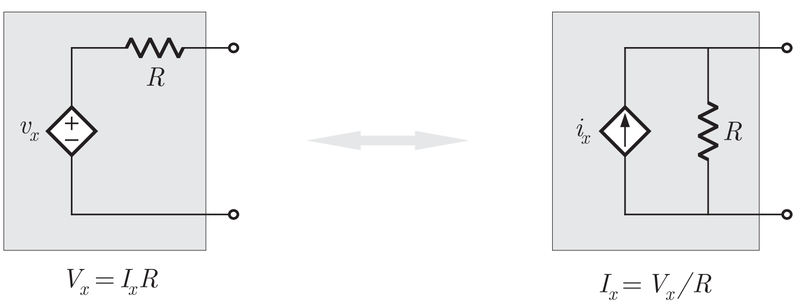
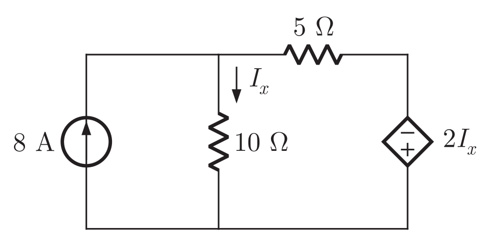


Fig 5.4.4 Source transformation of dependent sources

EXAMPLE

The value of current I_x in the following circuit, is equal to



(A) 3.0 A (B) 4.21 A

(C) 2.35 A (D) 6.15 A

SOLUTION :

We know that source transformation is also applicable to dependent sources. So, we transform the dependent voltage source into equivalent dependent current source

Now using current division

$$I_x = \frac{5}{10+5}\left(8 - \frac{2}{5}I_x\right)$$

$$I_x = \frac{1}{3}\left(8 - \frac{2}{5}I_x\right)$$

$$3I_x + \frac{2}{5}I_x = 8$$

$$\frac{17}{5}I_x = 8 \Rightarrow I_x = 2.35 \text{ A}$$

5.5 THEVENIN'S THEOREM

It states that any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source, V_{Th}, in series with an equivalent resistance, R_{Th} as illustrated in the figure 5.5.1.

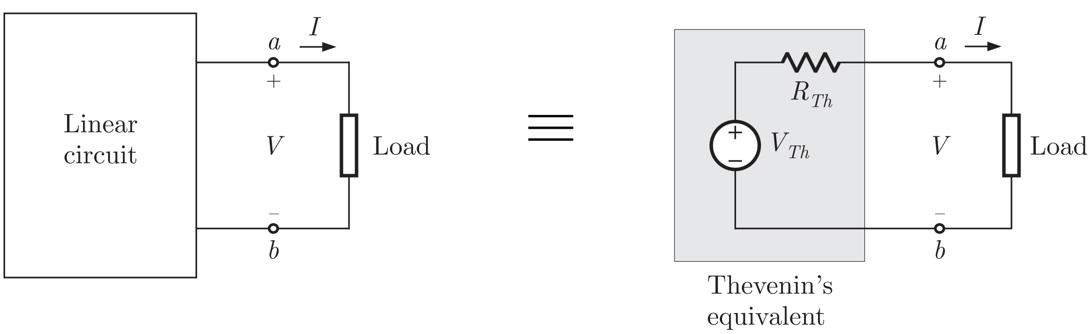


Fig 5.5.1 Illustration of Thevenin theorem

Where V_{Th} is called Thevenin's equivalent voltage or simply Thevenin voltage and R_{Th} is called Thevenin's equivalent resistance or simply Thevenin resistance.

The methods of obtaining Thevenin equivalent voltage and resistance are given in the following sections.

5.5.1 Thevenin's Voltage

The equivalent Thevenin voltage (V_{Th}) is equal to the open-circuit voltage present at the load terminals (with the load removed). Therefore, it is also denoted by V_{oc}

For the Thevenin voltage we may use the terms Thevenin voltage or open circuit voltage interchangeably.

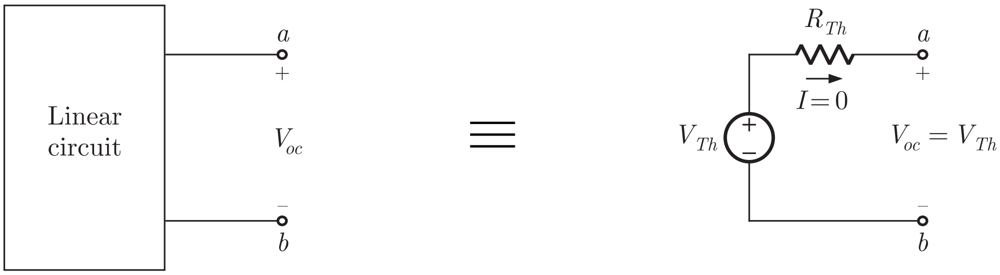


Fig 5.5.2 Equivalence of open circuit and Thevenin voltage

Figure 5.5.2 illustrates that the open-circuit voltage, V_{oc}, and the Thevenin voltage, V_{Th}, must be the same because in the circuit consisting of V_{Th} and R_{Th}, the voltage V_{oc} must equal V_{Th}, since no current flows through R_{Th} and therefore the voltage across R_{Th} is zero. Kirchhoff's voltage law confirms that

$$V_{Th} = R_{Th}(0) + V_{oc} = V_{oc}$$

The procedure of obtaining Thevenin voltage is given in the following methodology.

M E T H O D O L O G Y 1

1. Remove the load i.e open circuit the load terminals.
2. Define the open-circuit voltage V_{oc} across the open load terminals.
3. Apply any preferred method (KCL, KVL, nodal analysis, mesh analysis etc.) to solve for V_{oc}.
4. The Thevenin voltage is $V_{Th} = V_{oc}$.

Note that this methodology is applicable with the circuits containing both the dependent and independent source.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then its open circuit voltage or Thevenin voltage will simply be zero.

5.5.2 Thevenin's Resistance

Thevenin resistance is the input or equivalent resistance at the open circuit terminals a, b when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits).

We consider the following cases where Thevenin resistance R_{Th} is to be determined.

Case 1: Circuit With Independent Sources only

If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance or equivalent resistance of the network looking between terminals a and b, as shown in figure 5.5.3.

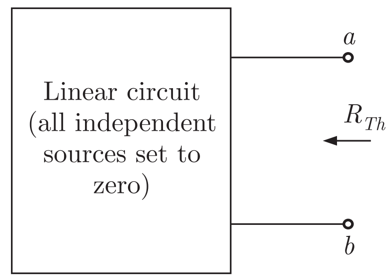


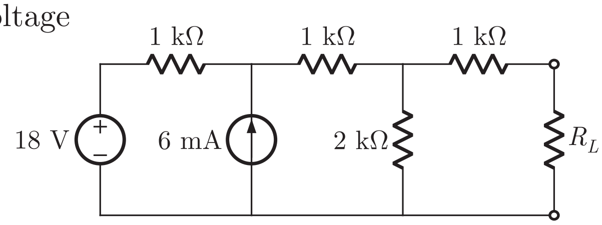
Fig 5.5.3 Circuit for obtaining R_{Th}

► EXAMPLE

In the circuit shown below, Thevenin equivalent voltage and resistance seen at load terminal, are equal to

(A) 6 V, 5 kΩ (B) 24 V, 5/3 kΩ

(C) 18 V, 1 kΩ (D) 12 V, 2 kΩ

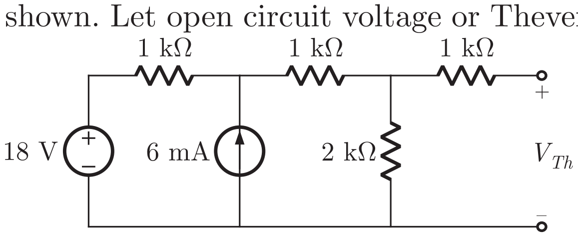


SOLUTION :

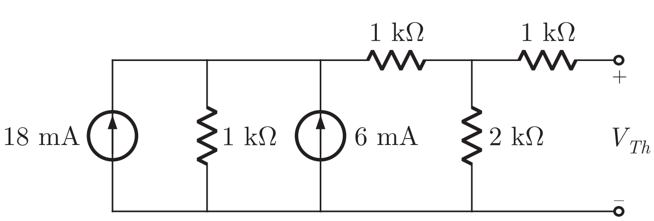
First we will find Thevenin equivalent across load terminals.

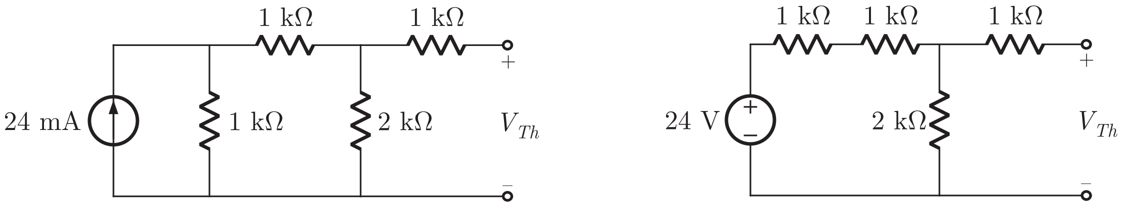
Thevenin voltage: (Open circuit voltage)

Remove the load R_L and open circuit its terminal as shown. Let open circuit voltage or Thevenin voltage is V_{Th},



Using source transformation



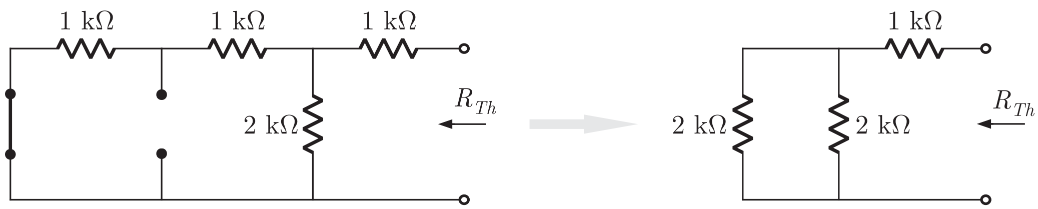


$$V_{Th} = \frac{2}{2+2}(24) \quad \text{(Using voltage division)}$$

$$= 12 \text{ V}$$

Thevenin resistance :

To obtain Thevenin resistance we set all independent sources to zero i.e. we short circuit the 18 V source and open circuit the 6 mA source as shown in figure below,



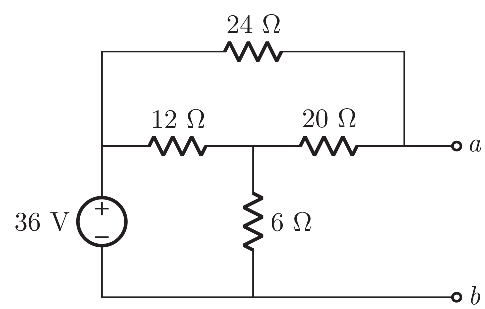
Thevenin resistance is the equivalent resistance seen at load terminals.

$$R_{Th} = 1 + 2\,||\,2 = 1 + 1 = 2 \text{ k}\Omega$$

Hence (D) is correct option.

▶ EXAMPLE

The Thevenin's equivalent of the circuit shown in the figure is



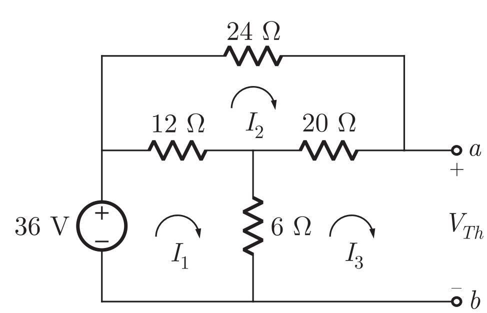
(A) 4 V, 48 Ω (B) 24 V, 12 Ω

(C) 24 V, 24 Ω (D) 12 V, 12 Ω

SOLUTION :

Thevenin voltage : (Open circuit voltage)

In the given problem, we use mesh analysis method to obtain Thevenin voltage



$$I_3 = 0 \qquad (a\text{-}b \text{ is open circuit})$$

Writing mesh equations

Mesh 1: $36 - 12(I_1 - I_2) - 6(I_1 - I_3) = 0$

$$36 - 12I_1 + 12I_2 - 6I_1 = 0 \qquad (I_3 = 0)$$

$$3I_1 - 2I_2 = 6 \qquad ...(i)$$

Mesh 2: $-24I_2 - 20(I_2 - I_3) - 12(I_2 - I_1) = 0$

$$-24I_2 - 20I_2 - 12I_2 + 12I_1 = 0 \qquad (I_3 = 0)$$

$$14I_2 = 3I_1 \qquad ...(ii)$$

From equation (i) and (ii)

$$I_1 = 7/3\text{ A}, \quad I_2 = 1/2\text{ A}$$

Mesh 3:

$$-6(I_3 - I_1) - 20(I_3 - I_2) - V_{Th} = 0$$

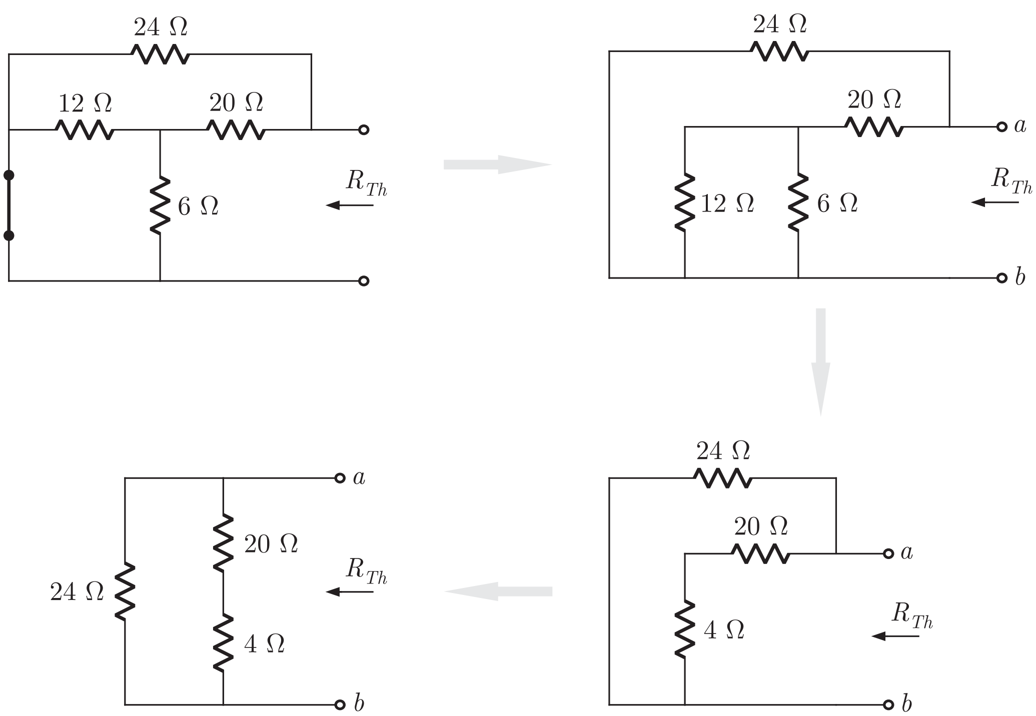
$$-6\left[0 - \frac{7}{3}\right] - 20\left[0 - \frac{1}{2}\right] - V_{Th} = 0$$

$$14 + 10 = V_{Th}$$

$$V_{Th} = 24\text{ volt}$$

Thevenin Resistance :

To obtain Thevenin resistance we set independent source to zero i.e. short circuit the 36 V source.



$$R_{Th} = (20 + 4) \,||\, 24\,\Omega$$

$$R_{Th} = 24\,\Omega \,||\, 24\,\Omega$$

$$R_{Th} = 12\,\Omega$$

Hence (B) is correct option.

► EXAMPLE

What values of R_{Th} and V_{Th} will cause the circuit of figure (B) to be the equivalent circuit of figure (A) ?

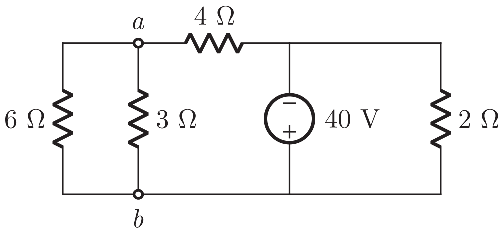


Fig.(A)

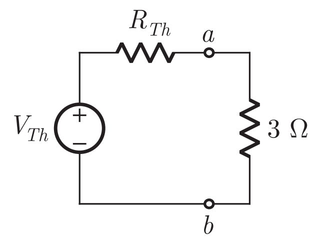


Fig.(B)

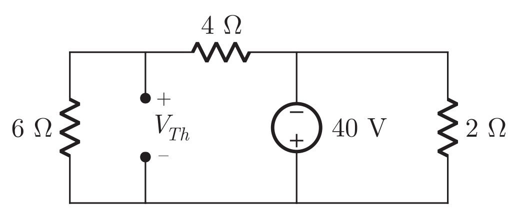
(A) 2.4 Ω, −24 V (B) 3 Ω, 16 V

(C) 10 Ω, 24 V (D) 10 Ω, −24 V

SOLUTION :

Thevenin voltage: (Open circuit voltage)

First we remove the load resistance (i.e open circuit the. 3 Ω resistance) and obtain the open circuit voltage across it.

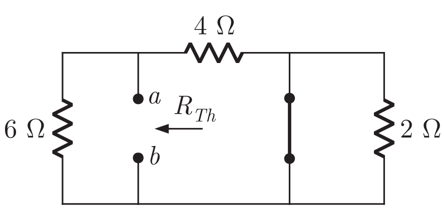


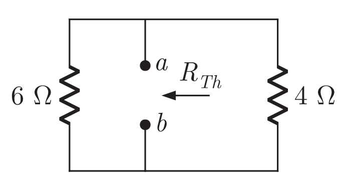
$$V_{Th} = \frac{6}{6+4}(-40) \qquad \text{(using voltage division)}$$

$$= -24 \text{ volt}$$

Thevenin resistance :

Set all independent source to zero(short circuit 40 V source)





$$R_{Th} = 6\,\Omega \,||\, 4\,\Omega = \frac{6\times 4}{6+4} = 2.4\,\Omega$$

Hence (A) is correct option.

Case 2: Circuit With Both Dependent and Independent Sources

Different methods can be used to determine Thevenin equivalent resistance of a circuit containing dependent sources. We may follow the given two methodologies. Both the methods are also applicable to circuit with independent sources only(case 1).

Using Test Source

METHODOLOGY 2

1. Set all independent sources to zero(Short circuit independent voltage source and open circuit independent current source).
2. Remove the load, and put a test source V_{test} across its terminals. Let the current through test source is I_{test}. Alternatively, we can put a test source I_{test} across load terminals and assume the voltage across it is V_{test}. Either method would give same result.
3. Thevenin resistance is given by $R_{Th} = V_{test}/I_{test}$.

We may use $V_{test} = 1\,\text{V}$ or $I_{test} = 1\,\text{A}$.

Using Short Circuit Current

$$R_{Th} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = \frac{V_{oc}}{I_{sc}}$$

METHODOLOGY 3

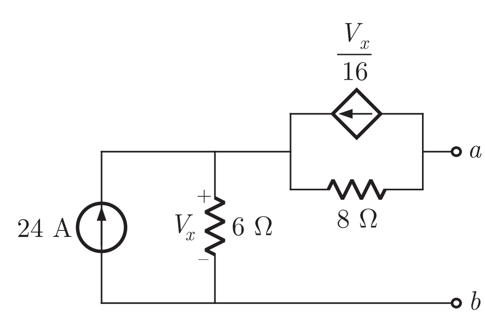
1. Connect a short circuit between terminal a and b.
2. Be careful, do not set independent sources zero in this method because we have to find short circuit current.
3. Now, obtain the short circuit current I_{sc} through terminals a, b.
4. Thevenin resistance is given as $R_{Th} = V_{oc}/I_{sc}$ where V_{oc} is open circuit voltage or Thevenin voltage across terminal a, b which can be obtained by same method given previously.

▶ EXAMPLE

The Thevenin equivalent resistance between terminal a and b in the following circuit is

(A) $22\,\Omega$ (B) $11\,\Omega$

(C) $17\,\Omega$ (D) $1\,\Omega$



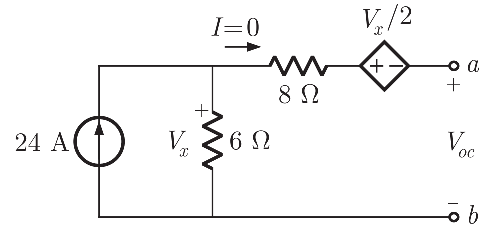
SOLUTION :

First we obtain R_{Th} using the methodology-3 which requires calculation of Thevenin voltage and short circuit current.

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{\text{Open circuit voltage}}{\text{short circuit current}}$$

Thevenin voltage: (Open circuit voltage V_{oc})

Using source transformation of the dependent source as shown in figure



Applying KCL at top left node

$$24 = \frac{V_x}{6} \Rightarrow V_x = 144 \text{ V}$$

Using KVL,

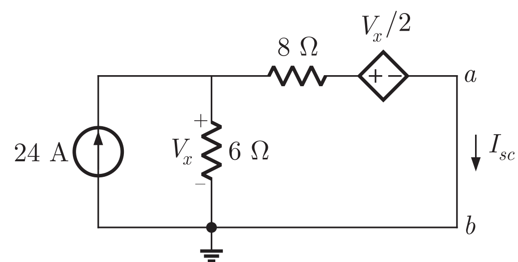
$$V_x - 8I - \frac{V_x}{2} - V_{oc} = 0$$

$$144 - 0 - \frac{144}{2} = V_{oc}$$

$$V_{oc} = 72 \text{ V}$$

Short circuit current (I_{sc}):

Applying KVL in the right mesh



$$V_x - 8I_{sc} - \frac{V_x}{2} = 0$$

$$\frac{V_x}{2} = 8I_{sc}$$

$$V_x = 16I_{sc}$$

Here V_x is different for both the cases because in first case terminals a, b are open circuit, while in the second case they are short circuited.

KCL at the top left node

$$24 = \frac{V_x}{6} + \frac{V_x - V_x/2}{8}$$

$$24 = \frac{V_x}{6} + \frac{V_x}{16}$$

$$V_x = \frac{1152}{11} \text{ V}$$

$$I_{sc} = \frac{V_x}{16} = \frac{1152}{11 \times 16} = \frac{72}{11} \text{ A}$$

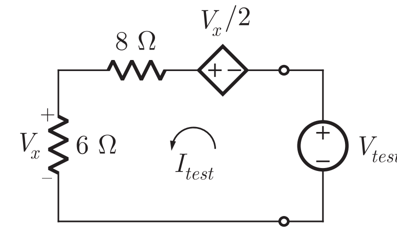
Thevenin resistance,

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{72}{\left(\frac{72}{11}\right)} = 11\,\Omega$$

Hence (B) is correct option.

Alternate method : (Methodology-2)

We can obtain Thevenin equivalent resistance without calculating the Thevenin voltage (open circuit voltage) as given in methodology-2. Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) and put a test source V_{test} between terminal a-b as shown



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

$$6I_{test} + 8I_{test} - \frac{V_x}{2} - V_{test} = 0 \qquad \text{(KVL)}$$

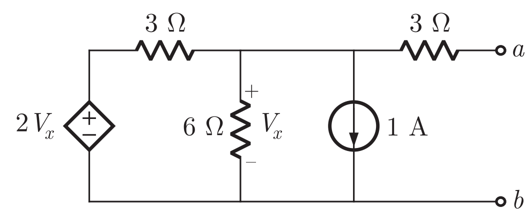
Thus $$14I - \frac{6I_{test}}{2} - V_{test} = 0 \qquad V_x = 6I_{test}$$

$$11I_{test} = V_{test}$$

So, $$R_{Th} = \frac{V_{test}}{I_{test}} = 11\,\Omega$$

▶ EXAMPLE

For the circuit shown in the figure, the Thevenin's voltage and resistance looking into a-b are



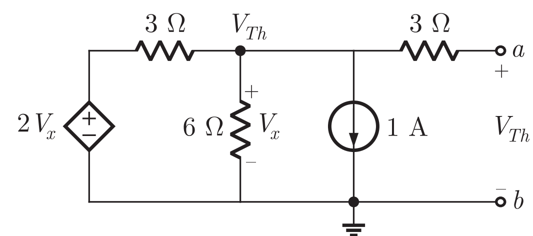
(A) 2 V, 3 Ω (B) 2 V, 2 Ω

(C) 6 V, −9 Ω (D) 6 V, −3 Ω

SOLUTION :

Thevenin voltage (Open circuit voltage) :

Applying KCL at top middle node



$$\frac{V_{Th} - 2V_x}{3} + \frac{V_{Th}}{6} + 1 = 0$$

$$\frac{V_{Th} - 2V_{Th}}{3} + \frac{V_{Th}}{6} + 1 = 0 \qquad (V_{Th} = V_x)$$

$$-2V_{Th} + V_{Th} + 6 = 0 \Rightarrow V_{Th} = 6 \text{ volt}$$

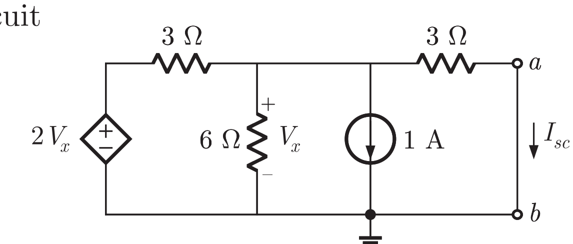
Thevenin Resistance :

To obtain Thevenin resistance we follow the procedure given in methodology-3.

$$R_{Th} = \frac{\text{Open circuit voltage}}{\text{Short circuit current}} = \frac{V_{Th}}{I_{sc}}$$

To obtain Thevenin resistance, first we find short circuit current through a-b

Writing KCL at top middle node



$$\frac{V_x - 2V_x}{3} + \frac{V_x}{6} + 1 + \frac{V_x - 0}{3} = 0$$

$$-2V_x + V_x + 6 + 2V_x = 0$$

$$V_x = -6 \text{ volt}$$

$$I_{sc} = \frac{V_x - 0}{3} = -\frac{6}{3} = -2\text{ A}$$

Thevenin's resistance, $$R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{6}{2} = -3\,\Omega$$

Hence (D) is correct option.

Circuit Analysis Using Thevenin Equivalent

Thevenin's theorem is very important in circuit analysis. It simplifies a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L, as shown in figure 5.5.5. The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained.

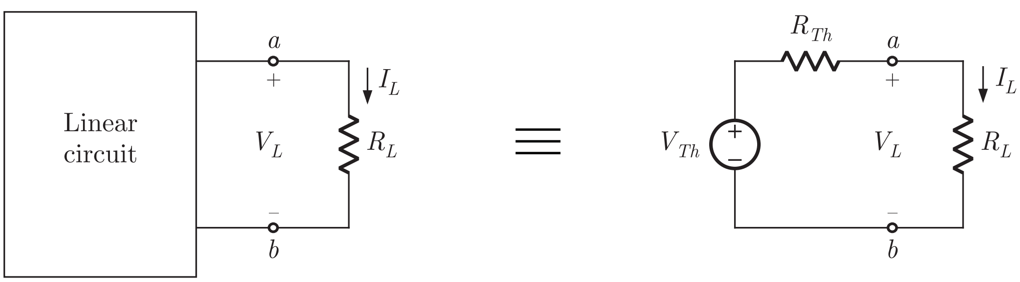


Fig 5.5.5 A circuit with a load and its equivalent Thevenin circuit

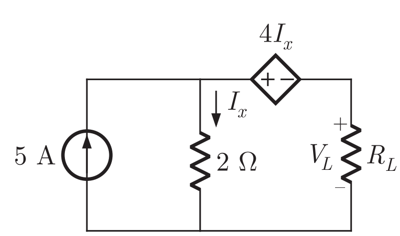
Current through the load R_L

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Voltage across the load R_L

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

▶ EXAMPLE



In the following circuit, if $R_L = 4\,\Omega$, then the voltage V_L across the load resistance R_L will be equal to

(A) -10 V (B) -20 V

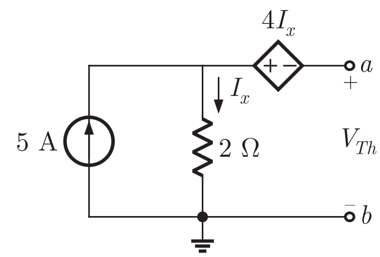
(C) 6.66 V (D) 40 V

SOLUTION :

We obtain Thevenin's equivalent across load terminal.

Thevenin voltage : (Open circuit voltage)

Follow the methodology-1 to calculate the Thevenin voltage across load resistance. Using KCL at top left node



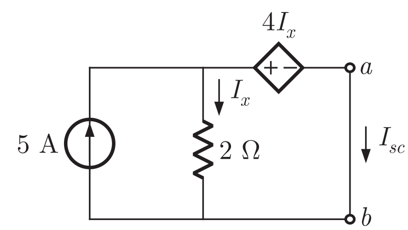
$$5 = I_x + 0$$
$$I_x = 5\text{ A}$$

$$2I_x - 4I_x - V_{Th} = 0 \qquad \text{(Using KVL)}$$
$$2(5) - 4(5) = V_{Th}$$
$$V_{Th} = -10\text{ volt}$$

Thevenin Resistance :

Follow the methodology-3 to obtain the Thevenin resistance. First we find short circuit current through a-b

Using KCL at top left node



$$5 = I_x + I_{sc}$$
$$I_x = 5 - I_{sc}$$

Applying KVL in the right mesh

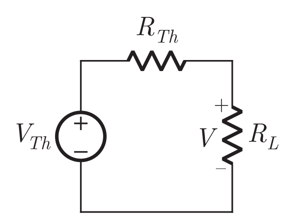
$$2I_x - 4I_x + 0 = 0$$
$$I_x = 0$$

So, $\quad 5 - I_{sc} = 0$ or $I_{sc} = 5\text{ A}$

Thevenin resistance,

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{10}{5} = -2\,\Omega$$

Now, the circuit becomes as shown in figure



$$V = V_{Th}\left(\frac{R_L}{R_{Th} + R_L}\right) \qquad \text{(Using voltage division)}$$
$$= (-10)\left(\frac{4}{-2+4}\right) \qquad R_L = 4\,\Omega$$
$$= -20\text{ V}$$

Hence (C) is correct option.

5.6 NORTON'S THEOREM

Any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source, I_N, in parallel with an equivalent resistance, R_N as illustrated in figure 5.6.1.

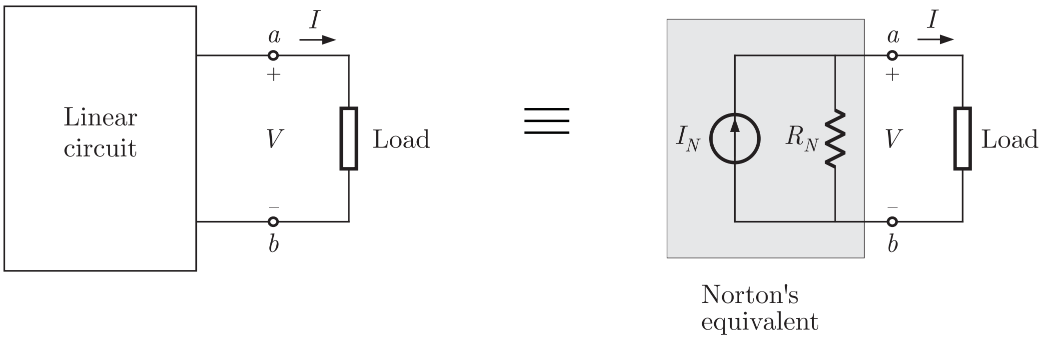


Fig 5.6.1 Illustration of Norton theorem

Where I_N is called Norton's equivalent current or simply Norton current and R_N is called Norton's equivalent resistance. The methods of obtaining Norton equivalent current and resistance are given in the following sections.

5.6.1 Norton's Current

The Norton equivalent current is equal to the short-circuit current that would flow when the load replaced by a short circuit. Therefore, it is also called short circuit current I_{sc}.

For the Norton current we may use the term Norton current or short circuit current interchangeably.

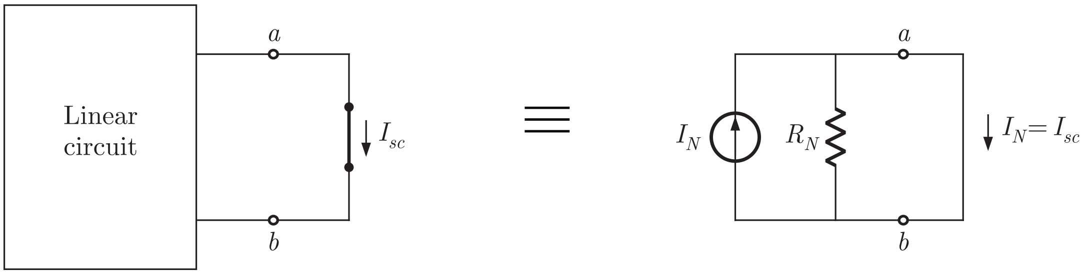


Fig 5.6.2 Equivalence of short circuit current and Norton current

Figure 5.6.2 illustrates that if we replace the load by a short circuit, then current flowing through this short circuit will be same as Norton current I_N

$$I_N = I_{sc}$$

The procedure of obtaining Norton current is given in the following methodology

METHODOLOGY

1. Replace the load with a short circuit.
2. Define the short circuit current, I_{sc}, through load terminal.
3. Obatin I_{sc} using any method (KCL, KVL, nodal analysis, loop analysis).
4. The Norton current is $I_N = I_{sc}$.

Note that this methodology is applicable with the circuits containing both the dependent and independent source.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then the short circuit current or Norton current will simply be zero.

5.6.2 Norton's Resistance

Norton resistance is the input or equivalent resistance seen at the load terminals when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits) i.e. Norton resistance is same as Thevenin's resistance

$$R_N = R_{Th}$$

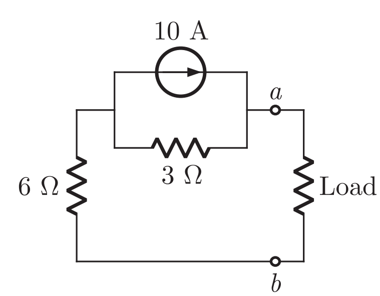
We may denote Norton resistance either by R_N or R_{Th}.

So, we can obtain Norton resistance using same methodologies as for Thevenin resistance. Dependent and independent sources are treated the same way as in Thevenin's theorem.

EXAMPLE

What are the values of equivalent Norton current source (I_N) and equivalent resistance (R_N) across the load terminal of the circuit shown in figure ?

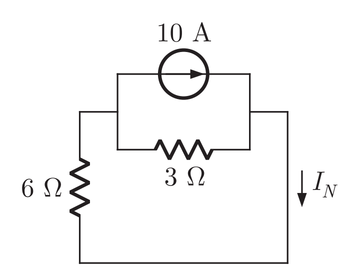
	I_N	R_N
(A)	10 A	2 Ω
(B)	10 A	9 Ω
(C)	3.33 A	9 Ω
(D)	6.66 A	2 Ω



SOLUTION :

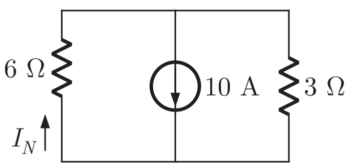
Norton Current(short circuit current):

Short circuit current across terminal a-b is obtained by



using the methodology.

For simplicity circuit can be redrawn as show below

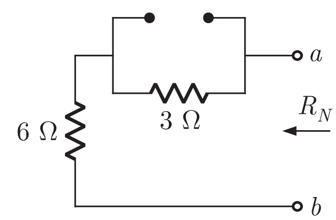


$$I_N = \frac{3}{3+6}(10) \qquad \text{(Current division)}$$

$$= 3.33 \text{ A}$$

Norton's equivalent resistance :

To obtain the Norton resistance, set independent source to zero(open circuit the 10 A current source)

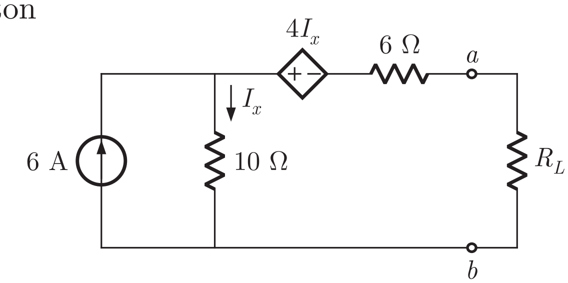


$$R_N = 6 + 3 = 9\,\Omega$$

Hence (C) is correct option.

► EXAMPLE

In the following circuit, what are the values of Norton equivalent current and resistance across terminal a-b ?



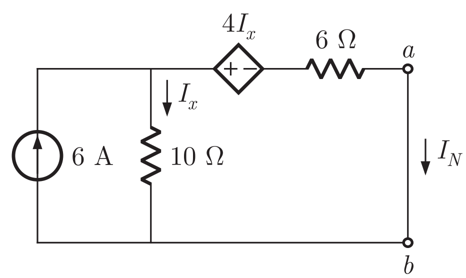
(A) 6 A, 12 Ω (B) 3 A, 12 Ω

(C) 2 A, 8 Ω (D) 3 A, 8 Ω

SOLUTION :

Norton current (short circuit current):

We replace load by a short circuit and calculate short circuit current.



KCL at the top left node

$$I_x + I_N = 6 \qquad \text{...(i)}$$

Writing KVL in the right mesh

$$10I_x - 4I_x - 6I_N = 0$$

$$6I_x = 6I_N$$

$$I_x = I_N$$

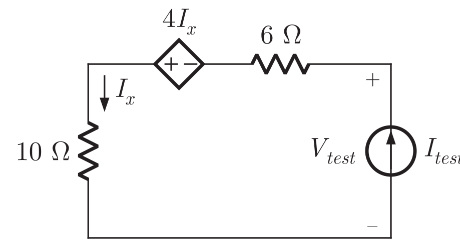
Substituting this into equation (i)

$$2I_N = 6$$

$$I_N = 3 \text{ A}$$

Norton equivalent resistance:

Since $R_N = R_{Th}$, we follow the same procedure given in methodology-2 as for R_{Th}. Dependent source is present in the network, therefore we put a test source across load terminal and set all independent sources to zero. (Open circuit 6 A source)



$$R_N = R_{Th} = \frac{V_{test}}{I_{test}}$$

By applying KVL

$$V_{test} - 6I_{test} + 4I_x - 10I_x = 0$$

$$V_{test} - 6I_{test} - 6I_x = 0$$

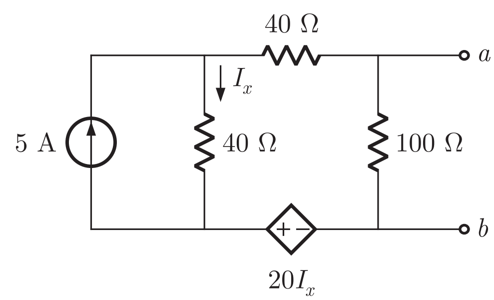
$$V_{test} - 6I_{test} - 6I_{test} = 0 \qquad (I_x = I_{test})$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = 12\,\Omega$$

Hence (B) is correct option.

▶ EXAMPLE

In the following circuit value of Norton current (I_N) and resistance (R_N) with respect to terminals a, b are



(A) $I_N = 5\text{ A},\ R_N = 50\,\Omega$

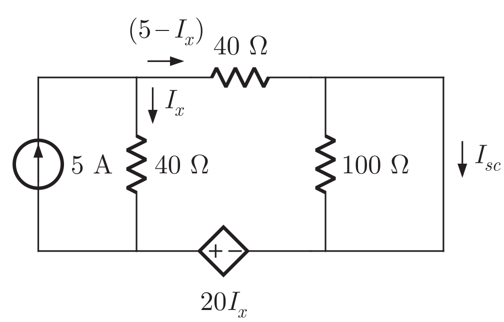
(B) $I_N = 3\text{ A},\ R_N = 50\,\Omega$

(C) $I_N = 9\text{ A},\ R_N = 100\,\Omega$

(D) $I_N = 6\text{ A},\ R_N = 150\,\Omega$

SOLUTION :

Norton current (Short circuit current)



By applying KVL

$$-(5 - I_x)40 - 0 + 20I_x + 40I_x = 0$$

$$-200 + 40I_x + 20I_x + 40I_x = 0$$

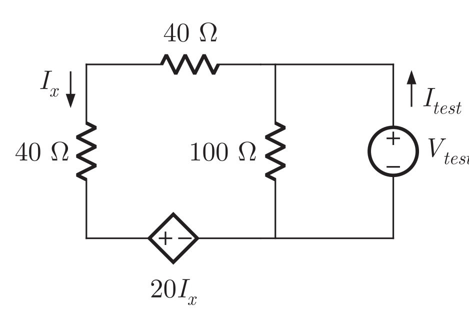
$$100I_x = 200$$

$$I_x = 2\text{ A}$$

$$I_{sc} = 5 - I_x = 5 - 2 = 3\text{ A}$$

Norton resistance

To obtain Norton resistance we set independent source to zero (open circuit 5 A source) and put a test source across a, b.



$$R_{Th} = R_N = \frac{V_{test}}{I_{test}}$$

Applying KVL

$$V_{test} - 40I_x - 40I_x - 20I_x = 0$$

$$V_{test} = 100I_x \qquad \text{...(i)}$$

Writing node equation at top right node

$$I_{test} = \frac{V_{test}}{100} + I_x$$

Substituting $I_x = V_{test}/100$ from equation (i)

$$I_{test} = \frac{V_{test}}{100} + \frac{V_{test}}{100}$$

$$I_{test} = \frac{V_{test}}{50}$$

$$R_{Th} = R_N = \frac{V_{test}}{I_{test}} = 50\,\Omega$$

Hence (B) is correct option.

Circuit Analysis Using Norton's Equivalent :

As discussed for Thevenin's theorem, Norton equivalent is also useful in circuit analysis. It simplifies a circuit. Consider a linear circuit terminated by a load R_L, as shown in figure 5.6.4. The current I_L through the load and the voltage V_L across the load are easily determined once the Norton equivalent of the circuit at the load's terminals is obtained,

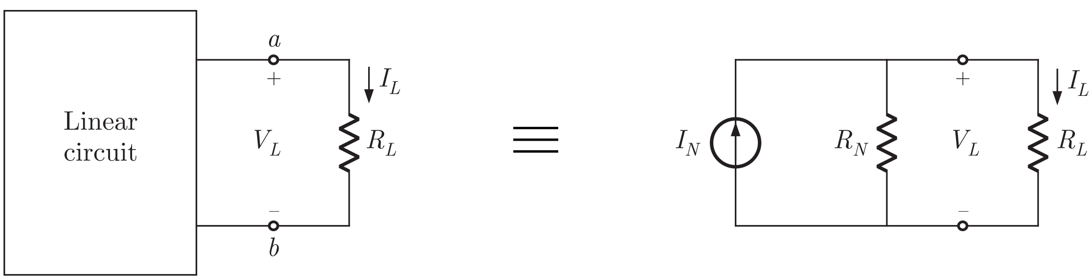


Fig 5.6.4 A circuit with a load and its equivalent Norton circuit

Current through load R_L is,

$$I_L = \frac{R_N}{R_L + R_L} I_N$$

Voltage across load R_L is,

$$V_L = R_L I_L = \frac{R_L R_N}{R_{Th} + R_L} I_N$$

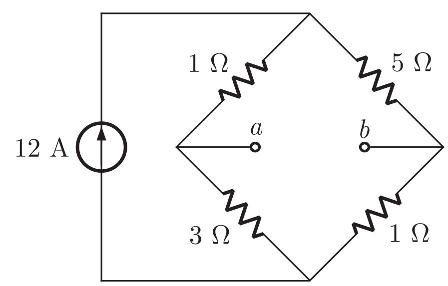
5.7 Transformation Between Thevenin & Norton's Equivalent Circuits

From source transformation it is easy to find Norton's and Thevenin's equivalent circuit from one form to another as following

Fig 5.7.1 Source transformation of Thevenin and Norton equivalents

► EXAMPLE

In the circuit, the Norton equivalent current with respect to terminal *a*-*b* is

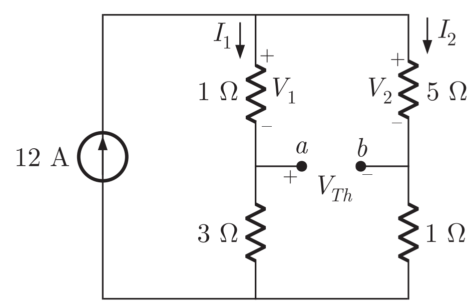


(A) 13 A (B) 7 A

(C) 8 A (D) 10 A

SOLUTION :

We obtain Thevenin's resistance across *a*-*b* and then use source transformation of Thevenin's circuit to obtain equivalent Norton circuit.



Using current division

$$I_1 = \frac{(5+1)}{(5+1)+(3+1)}(12) = \frac{6}{6+4}(12) = 7.2\text{ A}$$

$$V_1 = I_1 \times 1 = 7.2\text{ V}$$

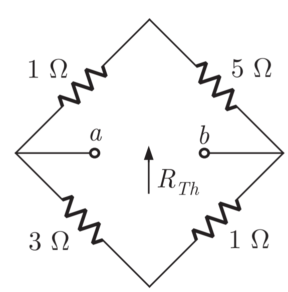
$$I_2 = \frac{(3+1)}{(3+1)+(5+1)}(12) = 4.8\text{ A}$$

$$V_2 = 5I_2 = 5 \times 4.8 = 24\text{ V}$$

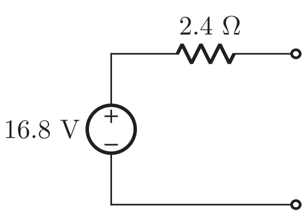
$$V_{Th} + V_1 - V_2 = 0 \qquad \text{(KVL)}$$

$$V_{Th} = V_2 - V_1 = 24 - 7.2 = 16.8\text{ V}$$

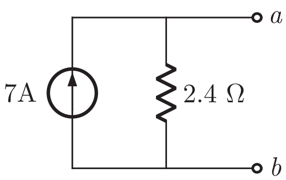
$$R_{Th} = (5+1)\,||\,(3+1) = 6\,||\,4 = 2.4\,\Omega$$



Thevenin's equivalent is



Norton equivalent using source transformation, is



Hence (B) is correct option.

5.8 MAXIMUM POWER TRANSFER THEOREM

Maximum power transfer theorem states that a load resistance R_L will receive maximum power from a circuit when the load resistance is equal to Thevenin's/Norton's resistance seen at load terminals.

i.e. $R_L = R_{Th}$, (For maximum power transfer)

In other words a network delivers maximum power to a load resistance R_L when R_L is equal to Thevenin equivalent resistance of the network.

Proof :

Consider the Thevenin equivalent circuit of figure 5.8.1 with Thevenin voltage V_{Th} and Thevenin resistance R_{Th}. We assume that we can adjust the load resistance R_L. The power absorbed by the load, P_L, is given by the expression

$$P_L = I_L^2 R_L \tag{5.8.1}$$

and that the load current is given as,

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} \tag{5.8.2}$$

Substituting I_L from equation (5.8.2) into equation (5.8.1)

$$P_L = \frac{V_{Th}^2}{(R_L + R_{Th})^2} R_L \tag{5.8.3}$$

To find the value of R_L that maximizes the expression for P_L (assuming that V_{Th} and R_{Th} are fixed), we write

$$\frac{dP_L}{dR_L} = 0$$

Computing the derivative, we obtain the following expression :

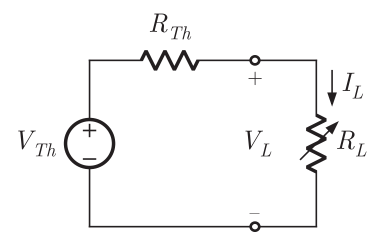


Fig 5.8.1 A circuit used for maximum power transfer

$$\frac{dP_L}{dR_L} = \frac{V_{Th}^2(R_L + R_{Th})^2 - 2V_{Th}^2 R_L(R_L + R_{Th})}{(R_L + R_{Th})^4}$$

which leads to the expression

$$(R_L + R_{Th})^2 - 2R_L(R_L + R_{Th}) = 0$$

or $$R_L = R_{Th}$$

Thus, in order to transfer maximum power to a load, the equivalent source and load resistances must be matched, that is, equal to each other.

$$R_L = R_{Th}$$

The maximum power transferred is obtained by substituting $R_L = R_{Th}$ into equation (5.8.3)

$$P_{\max} = \frac{V_{Th}^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_{Th}^2}{4R_{Th}} \qquad (4.24)$$

or, $$P_{\max} = \frac{V_{Th}^2}{4R_L}$$

If the Load resistance R_L is fixed :

Now consider a problem where the load resistance R_L is fixed and Thevenin resistance or source resistance R_s is being varied, then

$$P_L = \frac{V_{Th}^2}{(R_L + R_s)^2} R_L$$

To obtain maximum P_L denominator should be minimum or $R_s = 0$. This can be solved by differentiating the expression for the load power, P_L, with respect to R_s instead of R_L.

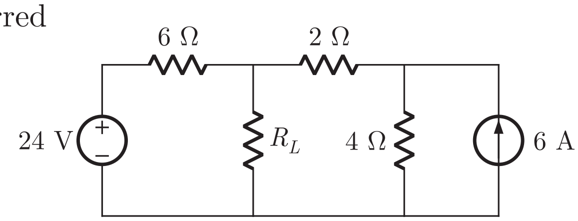
The step-by-step methodology to solve problems based on maximum power transfer is given as following :

METHODOLOGY

1. Remove the load R_L and find the Thevenin equivalent voltage V_{Th} and resistance R_{Th} for the remainder of the circuit.
2. Select $R_L = R_{Th}$, for maximum power transfer.
3. The maximum average power transfer can be calculated using $P_{\max} = V_{Th}^2/4R_{Th}$.

▶ EXAMPLE

In the circuit shown below the maximum power transferred to R_L is $P_{\max}$, then



(A) $R_L = 12\,\Omega,\ P_{\max} = 12\text{ W}$

(B) $R_L = 3\,\Omega,\ P_{\max} = 96\text{ W}$

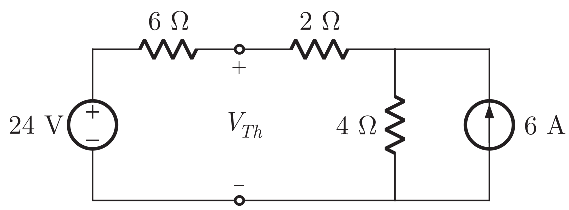
(C) $R_L = 3\,\Omega,\ P_{\max} = 48\text{ W}$

(D) $R_L = 12\,\Omega,\ P_{\max} = 24\text{ W}$

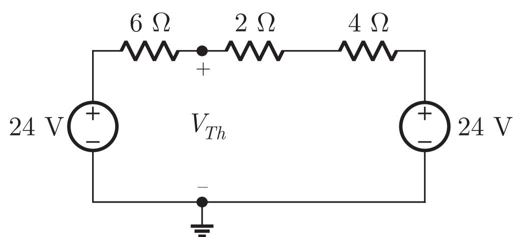
SOLUTION :

Step 1: First, obtain Thevenin equivalent across R_L.

Thevenin voltage : (Open circuit voltage)



Using source transformation

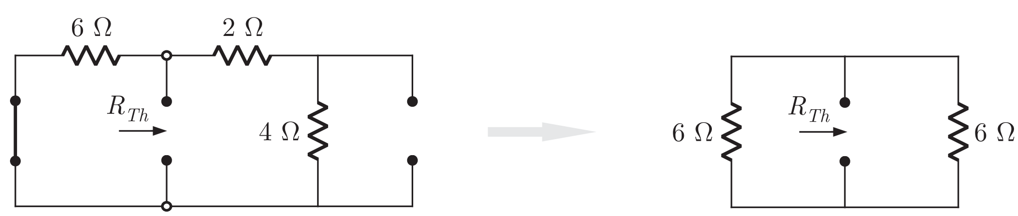


Using nodal analysis

$$\frac{V_{Th} - 24}{6} + \frac{V_{Th} - 24}{2 + 4} = 0$$

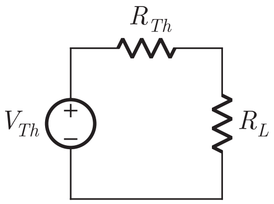
$$2V_{Th} - 48 = 0 \Rightarrow V_{Th} = 24\text{ V}$$

Thevenin resistance :



$$R_{Th} = 6\,\Omega\,||\,6\,\Omega = 3\,\Omega$$

Circuit becomes as



Step 2: For maximum power transfer

$$R_L = R_{Th} = 3\,\Omega$$

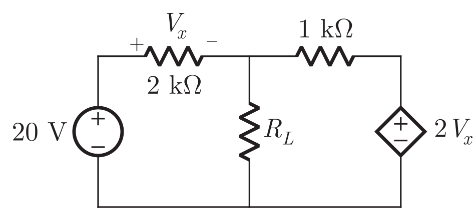
Step 3: Value of maximum power

$$P_{max} = \frac{(V_{Th})^2}{4R_L} = \frac{(24)^2}{4 \times 3} = 48 \text{ W}$$

Hence (C) is correct option.

► EXAMPLE

In the circuit shown, what value of R_L maximizes the power delivered to R_L ?

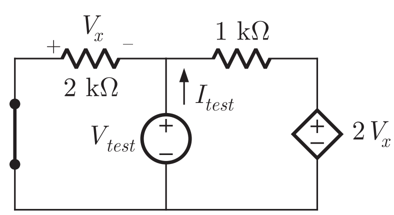


(A) $286\,\Omega$ (B) $350\,\Omega$

(C) zero (D) $500\,\Omega$

SOLUTION :

For maximum power transfer $R_L = R_{Th}$. To obtain Thevenin resistance set all independent sources to zero and put a test source across load terminals.



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Writing KCL at the top center node

$$\frac{V_{test}}{2\text{k}} + \frac{V_{test} - 2V_x}{1\text{k}} = I_{test} \quad \text{...(i)}$$

Also, $V_{test} + V_x = 0$ (KVL in left mesh)

So, $V_x = -V_{test}$

Substituting $V_x = -V_{test}$ into equation (i)

$$\frac{V_{test}}{2\text{k}} + \frac{V_{test} - 2(-V_{test})}{1\text{k}} = I_{test}$$

$$V_{test} + 6V_{test} = 2I_{test}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{2}{7}\text{ k}\Omega \simeq 286\,\Omega$$

Hence (A) is correct option.

5.9 Reciprocity Theorem

The reciprocity theorem is a theorem which can only be used with single source circuits (either voltage or current source). The theorem states the following

Circuit With a Voltage Source

In any linear bilateral network, if a single voltage source V_a in branch a produces a current I_b in another branch b, then if the voltage source V_a is removed(i.e. short circuited) and inserted in branch b, it will produce a current I_b in branch a.

In other words, it states that the ratio of response(output) to excitation(input) remains constant if the positions of output and input are interchanged in a reciprocal network. Consider the network shown in figure 5.9.1a and b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \tag{5.9.1}$$

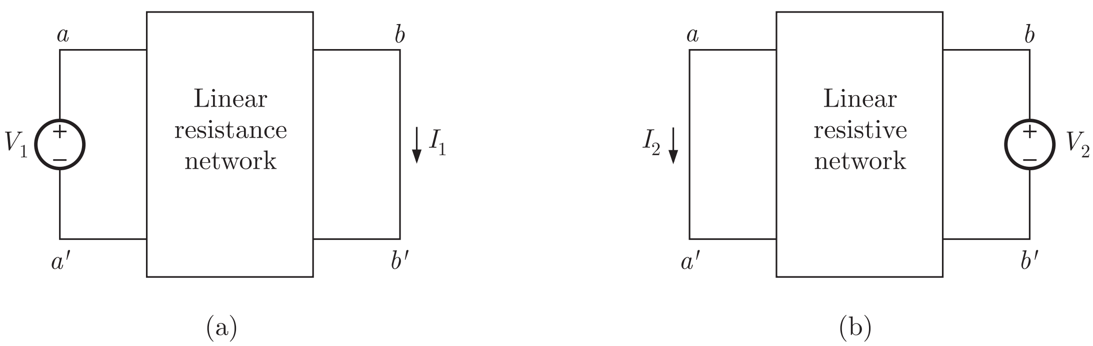


Fig 5.9.1 Illustration of reciprocity theorem for a voltage source

When applying the reciprocity theorem for a voltage source, the following steps must be followed:

1. The voltage source is replaced by a short circuit in the original location.
2. The polarity of the voltage source in the new location have the same correspondence with branch current, in each position, otherwise a −ve sign appears in the expression (5.9.1).

This can be explained in a better way through following example.

▶ EXAMPLE

In the circuit of figure (A), if $I_1 = 20\,\text{mA}$, then what is the value of current I_2 in the circuit of figure (B) ?

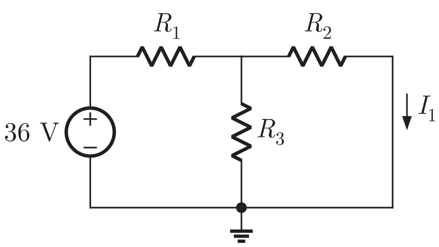


Fig.(A)

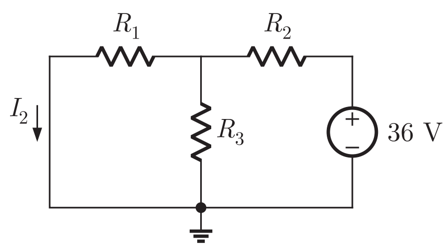


Fig.(B)

(A) 40 mA

(B) −20 mA

(C) 20 mA

(D) R_1, R_2 and R_3 must be known

SOLUTION :

In figure (A), $V_1 = 36\,\text{V},\ \ I_1 = 20\,\text{mA}$

In figure (B), $V_2 = 36\,\text{V},\ \ I_2 = ?$

Using reciprocity

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

So, $I_2 = I_1 = 20\,\text{mA}$

Hence (C) is correct option.

▶ EXAMPLE

In the circuit shown in fig (a) if current $I_1 = 2.5\,\text{A}$ then current I_2 and I_3 in fig (B) and (C) respectively are

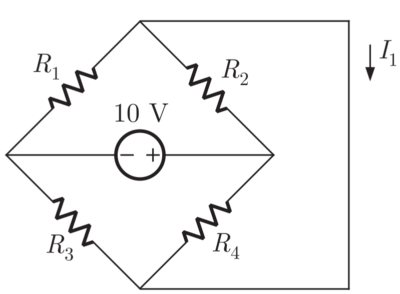


Fig.(A)

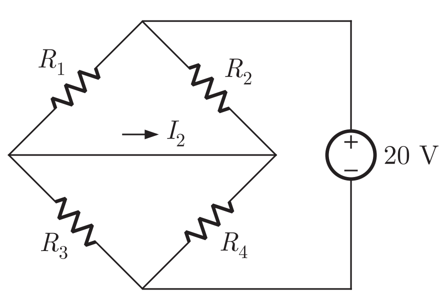


Fig.(B)

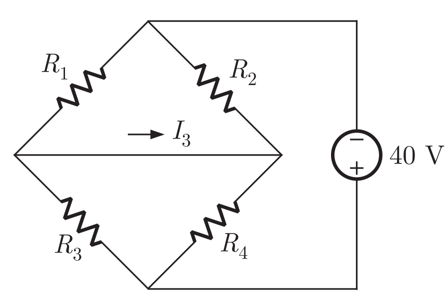


Fig.(C)

(A) 5 A, 10 A (B) −5 A, 10 A

(C) 5 A, −10 A (D) −5 A, −10 A

SOLUTION :

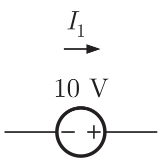


Fig.(A)

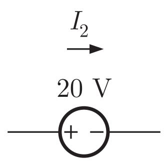


Fig.(B)

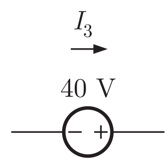


Fig.(C)

It can be solved by reciprocity theorem. Polarity of voltage source should have same correspondence with branch current in each of the circuit. Polarity of voltage source and current direction are shown below

So, $$\frac{V_1}{I_1} = -\frac{V_2}{I_2} = \frac{V_3}{I_3}$$

$$\frac{10}{2.5} = -\frac{20}{I_2} = \frac{40}{I_3}$$

$$I_2 = -5\text{ A}$$

$$I_3 = 10\text{ A}$$

Hence (B) is correct option.

Circuit With a Current Source

In any linear bilateral network, if a single current source I_a in branch a produces a voltage V_b in another branch b, then if the current source I_a is removed(i.e. open circuited) and inserted in branch b, it will produce a voltage V_b in open-circuited branch a.

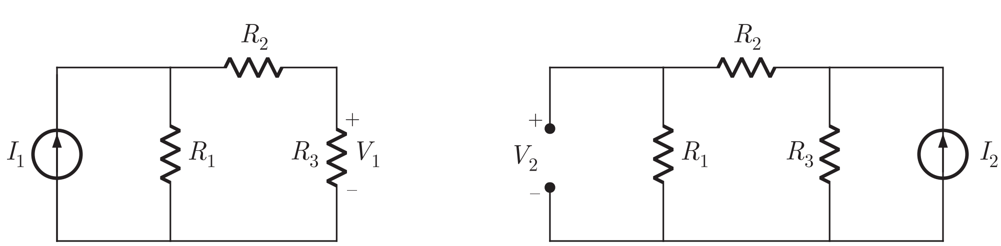


Fig 5.9.2 Illustration of reciprocity theorem for a current source

Again, the ratio of voltage and current remains constant. Consider the network shown in figure 5.9.2a and 5.9.2b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \tag{5.9.2}$$

When applying the reciprocity theorem for a current source, the following conditions must be met:

1. The current source is replaced by an open circuit in the original location.
2. The direction of the current source in the new location have the same correspondence with voltage polarity, in each position, otherwise a −ve sign appears in the expression (5.9.2).

Again the following example illustrated the above concepts using a better approach

► EXAMPLE

If $V_1 = 2$ V in the circuit of figure (A), then what is the value of V_2 in the circuit of figure (B) ?

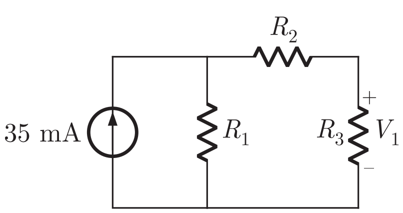


Fig.(A)

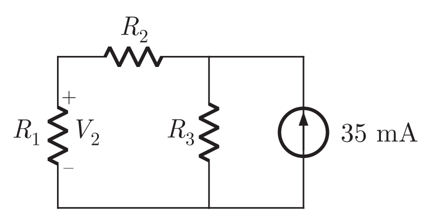


Fig.(B)

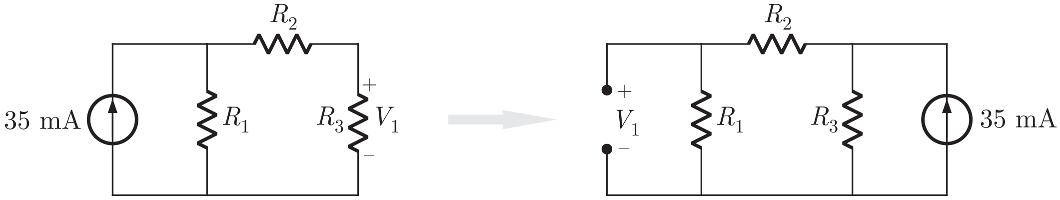
(A) 2 V

(B) −2 V

(C) 4 V

(D) R_1, R_2 and R_3 must be known

SOLUTION :



In figure (A), $I_1 = 35\,\text{mA}, \quad V_1 = 2\,\text{V}$

In figure (B), $I_2 = 35\,\text{mA}, \quad V_2 = ?$

Using reciprocity

$$\frac{V_1}{I_1} = \frac{V_2}{I_2}$$

So, $V_2 = V_1 = 2\,\text{volt}$

Hence (A) is correct option.

5.10 Substitution Theorem

If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

For example consider the circuit of figure 5.10.1 .The voltage V_{ab} and the current I in the circuit are given as

$$V_{ab} = \left(\frac{6}{6+4}\right)10 = 6\,\text{V}$$

$$I = \frac{10}{6+4} = 1\,\text{A}$$

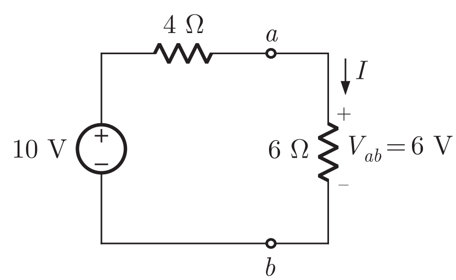


Fig 5.10.1 A circuit having voltage $V_{ab}=6$ V and current $I=1$ A in branch ab

The $6\,\Omega$ resistor in branch a-b may be replaced with any combination of components, provided that the terminal voltage and current must be the same.

We see that the branches of figure 5.10.2a-e are each equivalent to the original branch between terminals a and b of the circuit in figure 5.10.1.

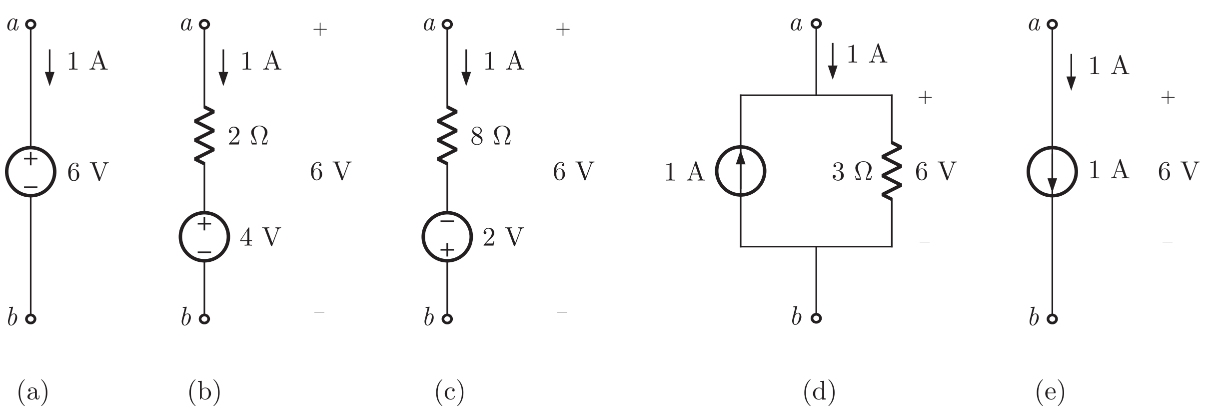


Fig 5.10.2 Equivalent circuits for branch ab

Also consider that the response of the remainder of the circuit of figure 5.10.1 is unchanged by substituting any one of the equivalent branches.

▶ EXAMPLE

If the $60\,\Omega$ resistance in the circuit of figure (A) is to be replaced with a current source I_s and $240\,\Omega$ shunt resistor as shown in figure (B), then magnitude and direction of required current source would be

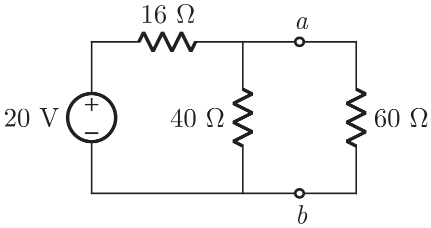


Fig.(A)

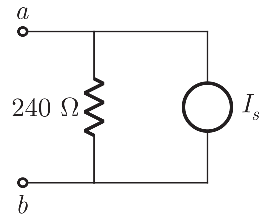


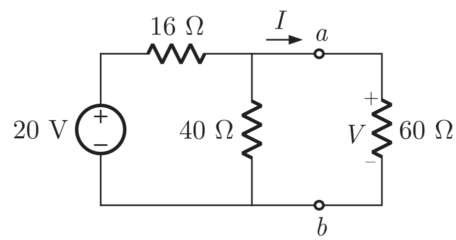
Fig.(B)

(A) 200 mA, upward (B) 150 mA, downward

(C) 50 mA, downward (D) 150 mA, upward

SOLUTION :

First we find the voltage and current for the branch ab, then substitute it with an equivalent.



$$V = \frac{40\,||\,60}{(40\,||\,60)+16}(20) \qquad \text{(using voltage division)}$$

$$= \frac{24}{40} \times 20 = 12\text{ V}$$

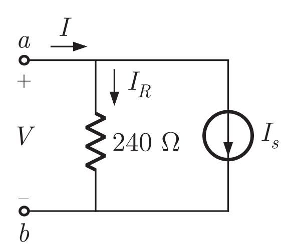
Current entering terminal a-b is

$$I = \frac{V}{R} = \frac{12}{60} = 200\text{ mA}$$

In fig(B), to maintain same voltage $V = 12\text{ V}$ current through $240\,\Omega$ resistor must be

$$I_R = \frac{12}{240} = 50\text{ mA}$$

Using KCL at terminal a, as shown



$$I = I_R + I_s$$

$$200 = 50 + I_s$$

$$I_s = 150\text{ mA}, \qquad \text{(down wards)}$$

Hence (B) is correct option.

5.11 MILLMAN'S THEOREM

Millman's theorem is used to reduce a circuit that contains several branches in parallel where each branch has a voltage source in series with a resistor as shown in figure.

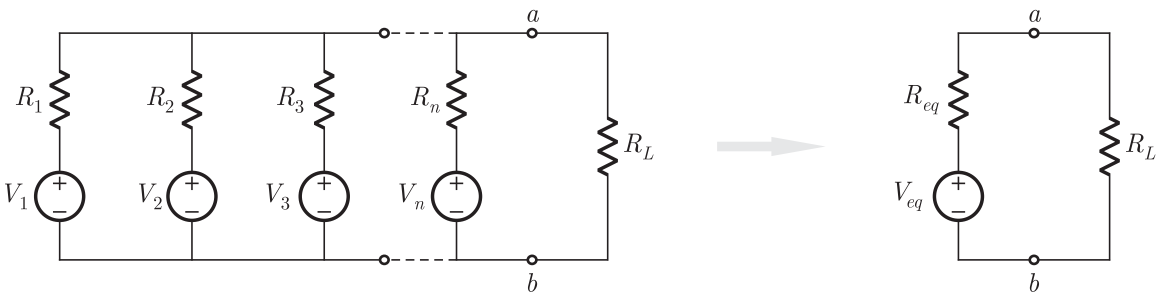


Fig 5.11.1 Illustration of Millman's Theorem

Mathematically

$$V_{eq} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + V_4 G_4 + \dots + V_n G_n}{G_1 + G_2 + G_3 + G_4 + \dots + G_n}$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + G_3 + \dots + G_n}$$

where conductances

$$G_1 = \frac{1}{R_1},\ G_2 = \frac{1}{R_2},\ G_3 = \frac{1}{R_3},\ G_4 = \frac{1}{R_4},\ \dots\ G_n = \frac{1}{R_n}$$

In terms of resistances

$$V_{eq} = \frac{V_1/R_1 + V_2/R_2 + V_3/R_3 + V_4/R_4 + \dots + V_n R_n}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + \dots + 1/R_n}$$

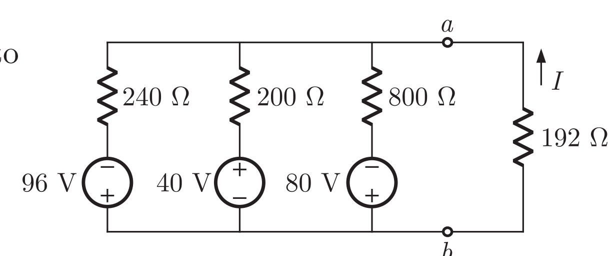
$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \dots + 1/R_n}$$

► EXAMPLE

The value of current I in the circuit below is equal to

(A) 100 mA (B) 10 mA

(C) 233.34 mA (D) none of these



SOLUTION :

We use Millman's theorem to obtain equivalent resistance and voltage across a-b.

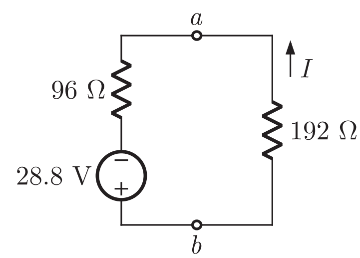
$$V_{ab} = \frac{-\frac{96}{240} + \frac{40}{200} + \frac{-80}{800}}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}}$$

$$= -\frac{144}{5} = -28.8\text{ V}$$

The equivalent resistance

$$R_{ab} = \frac{1}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}} = 96\,\Omega$$

Now, the circuit is reduced as shown in figure.



$$I = \frac{28.8}{96 + 192} = 100\text{ mA}$$

Hence (A) is correct option.

5.12 Tellegen's Theorem

Tellegen's theorem states that the sum of the power dissipations in a lumped network at any instant is always zero. This is supported by Kirchhoff's voltage and current laws. Tellegen's theorem is valid for any lumped network which may be linear or non-linear, passive or active, time-varying or time-invariant.

For a network with n branches, the power summation equation is,

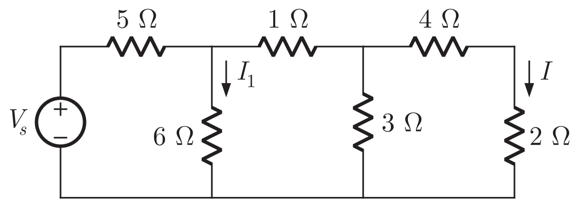
$$\sum_{k=1}^{k=n} V_k I_k = 0$$

One application of Tellegen's theorem is checking the quantities obtained when a circuit is analyzed. If the individual branch power dissipations do not add up to zero, then some of the calculated quantities are incorrect.

****

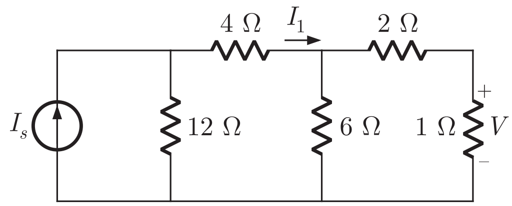
PRACTICE A

MCQ 5.1.1 In the network of figure for $V_s = V_0$, $I = 1$ A then what is the value of I_1, if $V_s = 2V_0$?



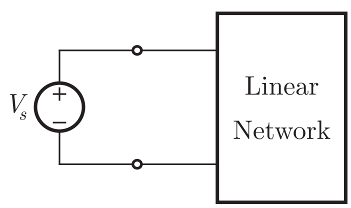
(A) 2 A
(B) 1.5 A
(C) 3 A
(D) 2.5 A

MCQ 5.1.2 In the network of figure, If $I_s = I_0$ then $V = 1$ V. What is the value of I_1 if $I_s = 2I_0$?



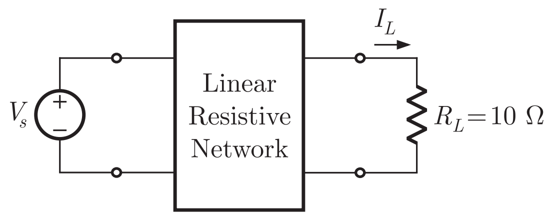
(A) 1.5 A
(B) 2 A
(C) 4.5 A
(D) 3 A

MCQ 5.1.3 The linear network in the figure contains resistors and dependent sources only. When $V_s = 10$ V, the power supplied by the voltage source is 40 W. What will be the power supplied by the source if $V_s = 5$ V ?



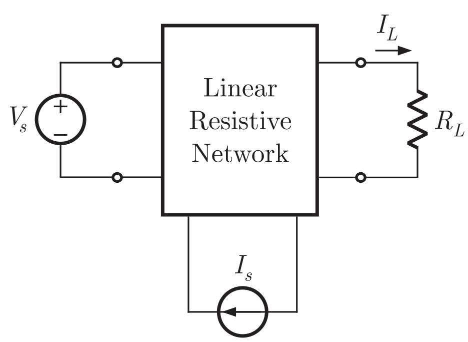
(A) 20 W
(B) 10 W
(C) 40 W
(D) can not be determined

MCQ 5.1.4 In the circuit below, it is given that when $V_s = 20\text{ V}$, $I_L = 200\text{ mA}$. What values of I_L and V_s will be required such that power absorbed by R_L is 2.5 W ?



(A) 1 A, 2.5 V
(B) 0.5 A, 2 V
(C) 0.5 A, 50 V
(D) 2 A, 1.25 V

MCQ 5.1.5 For the circuit shown in figure below, some measurements are made and listed in the table.

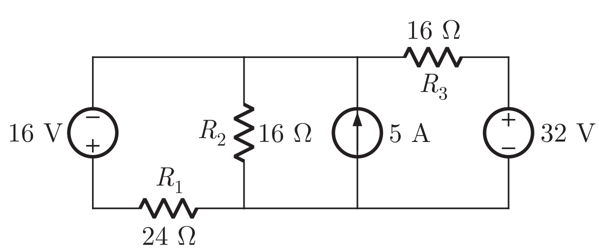


	V_s	I_s	I_L
1.	14 V	6 A	2 A
2.	18 V	2 A	6 A

Which of the following equation is true for I_L ?

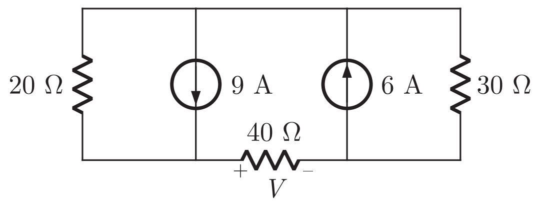
(A) $I_L = 0.6V_s + 0.4I_s$
(B) $I_L = 0.2V_s - 0.3I_s$
(C) $I_L = 0.2V_s + 0.3I_s$
(D) $I_L = 0.4V_s - 0.6I_s$

MCQ 5.1.6 In the circuit below, the voltage drop across the resistance R_2 will be equal to



(A) 46 volt
(B) 38 volt
(C) 22 volt
(D) 14 volt

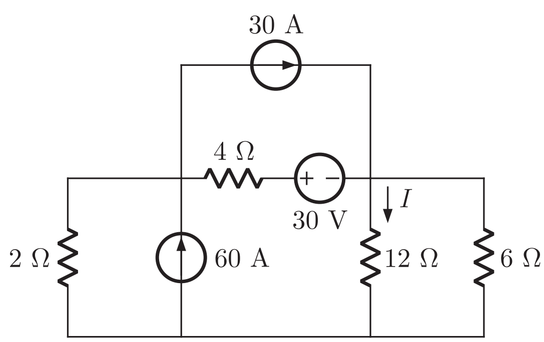
MCQ 5.1.7 In the circuit below, the voltage V across the $40\,\Omega$ resistor would be equal to



(A) 80 volt (B) 40 volt

(C) 160 volt (D) zero

MCQ 5.1.8 In the circuit below, current $I = I_1 + I_2 + I_3$, where I_1, I_2 and I_3 are currents due to 60 A, 30 A and 30 V sources acting alone. The values of I_1, I_2 and I_3 are respectively



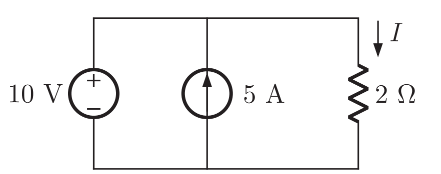
(A) 8 A, 8 A, −4 A

(B) 12 A, 12 A, −5 A

(C) 4 A, 4 A, −1 A

(D) 2 A, 2 A, −4 A

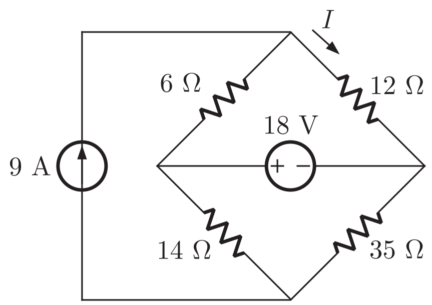
MCQ 5.1.9 The value of current I flowing through $2\,\Omega$ resistance in the circuit below, equals to



(A) 10 A (B) 5 A

(C) 4 A (D) zero

MCQ 5.1.10 In the circuit below, current I is equal to sum of two currents I_1 and I_2. What are the values of I_1 and I_2 ?

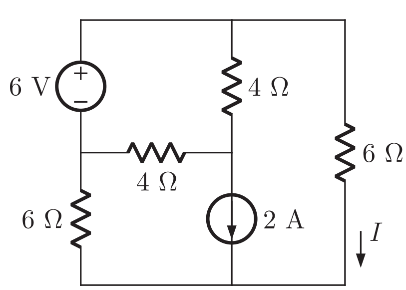


(A) 6 A, 1 A
(B) 9 A, 6 A
(C) 3 A, 1 A
(D) 3 A, 4 A

MCQ 5.1.11 A network consists only of independent current sources and resistors. If the values of all the current sources are doubled, then values of node voltages
(A) remains same
(B) will be doubled
(C) will be halved
(D) changes in some other way.

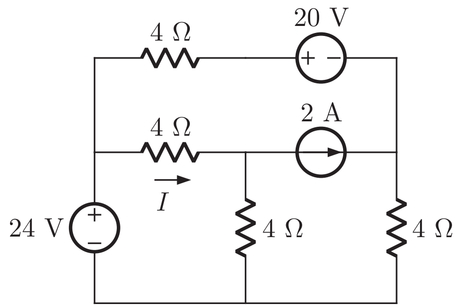
MCQ 5.1.12 Consider a network which consists of resistors and voltage sources only. If the values of all the voltage sources and doubled, then the values of mesh current will be
(A) doubled
(B) same
(C) halved
(D) none of these

MCQ 5.1.13 In the circuit shown in the figure below, the value of current I will be be given by



(A) 1.5 A (B) −0.3 A
(C) 0.05 A (D) −0.5 A

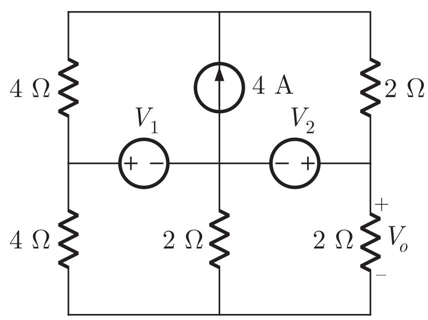
MCQ 5.1.14 What is the value of current I in the following network ?



(A) 4 A (B) 6 A

(C) 2 A (D) 1 A

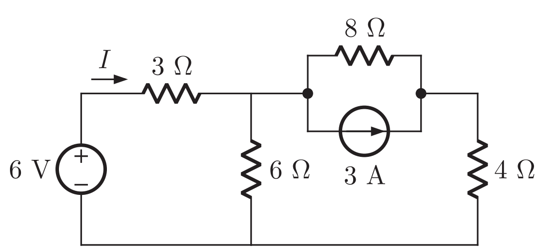
MCQ 5.1.15 In the given network if $V_1 = V_2 = 0$, then what is the value of V_o ?



(A) 3.2 V (B) 8 V

(C) 5.33 V (D) zero

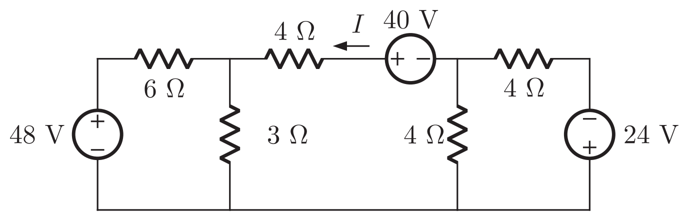
MCQ 5.1.16 The value of current I in the circuit below is equal to



(A) $\frac{2}{7}$ A (B) 1 A

(C) 2 A (D) 4 A

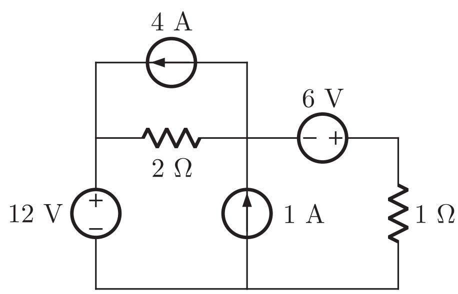
MCQ 5.1.17 What is the value of current I in the circuit shown below ?



(A) 8.5 A (B) 4.5 A

(C) 1.5 A (D) 5.5 A

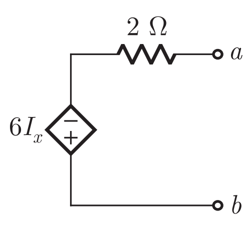
MCQ 5.1.18 In the circuit below, the 12 V source

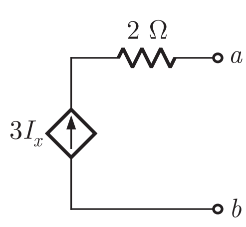


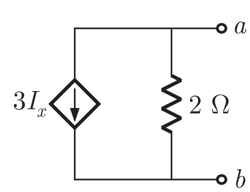
(A) absorbs 36 W (B) delivers 4 W

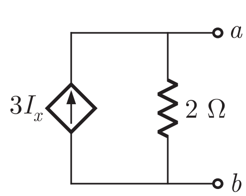
(C) absorbs 100 W (D) delivers 36 W

MCQ 5.1.19 Which of the following circuits in equivalent to the circuit shown below ?



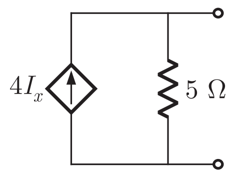
(A) 

(B) 

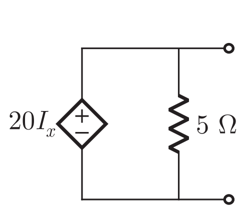
(C) 

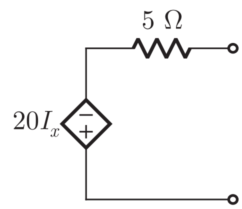
(D) None of these

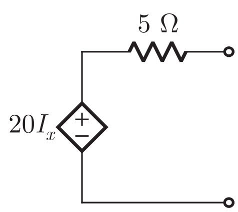
MCQ 5.1.20 Consider a dependent current source shown in figure below.



The source transformation of above is given by

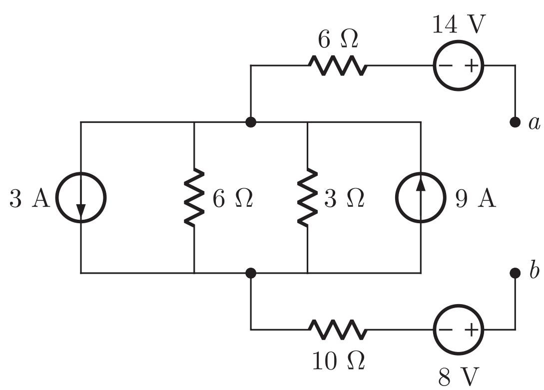
(A) 

(B) 

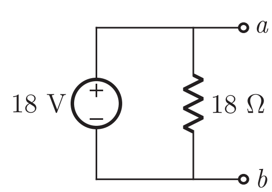
(C) 

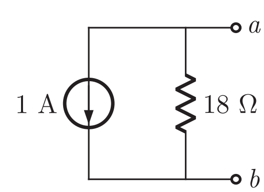
(D) Source transformation does not applicable to dependent sources

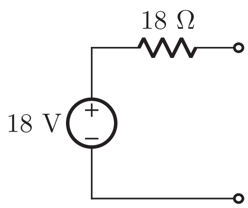
MCQ 5.1.21 Consider a circuit shown in the figure

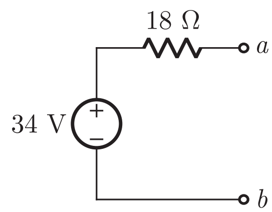


Which of the following circuit is equivalent to the above circuit ?

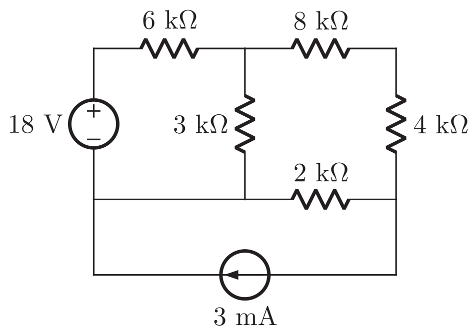
(A) 

(B) 

(C) 

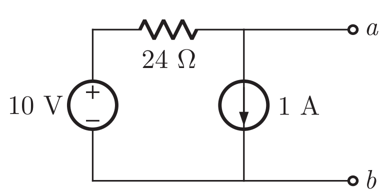
(D) 

MCQ 5.1.22 How much power is being dissipated by the $4\,k\Omega$ resistor in the network ?



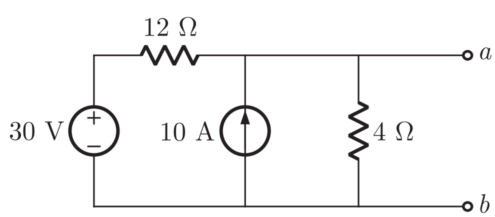
(A) 0 W
(B) 2.25 mW
(C) 9 mW
(D) 4 mW

MCQ 5.1.23 For the circuit shown in the figure the Thevenin voltage and resistance seen from the terminal a-b are respectively



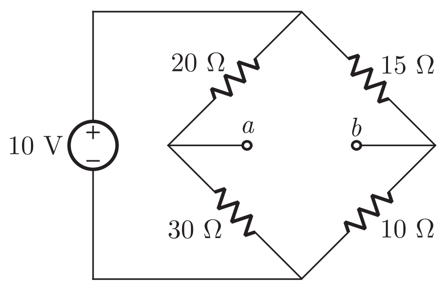
(A) $34\,V$, $0\,\Omega$
(B) $20\,V$, $24\,\Omega$
(C) $14\,V$, $0\,\Omega$
(D) $-14\,V$, $24\,\Omega$

MCQ 5.1.24 The Thevenin equivalent resistance R_{Th} between the nodes a and b in the following circuit is



(A) $3\,\Omega$
(B) $16\,\Omega$
(C) $12\,\Omega$
(D) $4\,\Omega$

MCQ 5.1.25 In the following circuit, Thevenin voltage and resistance across terminal a and b respectively are



(A) 10 V, 18 Ω (B) 2 V, 18 Ω

(C) 10 V, 18.67 Ω (D) 2 V, 18.67 Ω

MCQ 5.1.26 The value of R_{Th} and V_{Th} such that the circuit of figure (B) is the Thevenin equivalent circuit of the circuit shown is figure (A), will be equal to

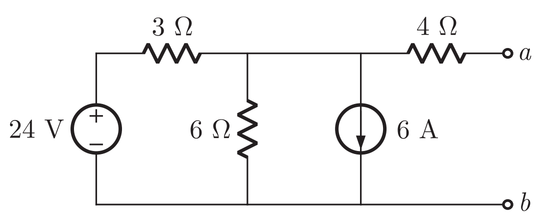


Fig.(A)

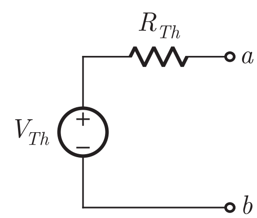


Fig.(B)

(A) $R_{Th} = 6\,\Omega$, $V_{Th} = 4$ V (B) $R_{Th} = 6\,\Omega$, $V_{Th} = 28$ V

(C) $R_{Th} = 2\,\Omega$, $V_{Th} = 24$ V (D) $R_{Th} = 10\,\Omega$, $V_{Th} = 14$ V

MCQ 5.1.27 What values of R_{Th} and V_{Th} will cause the circuit of figure (B) to be the equivalent circuit of figure (A) ?

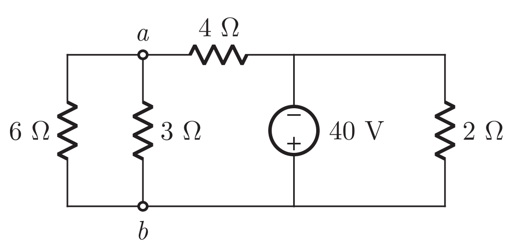


Fig.(A)

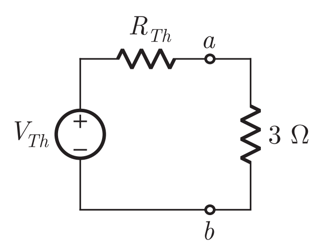


Fig.(B)

(A) 2.4 Ω, −24 V (B) 3 Ω, 16 V

(C) 10 Ω, 24 V (D) 10 Ω, −24 V

Common Data for Q. 34 to 35 :

Consider the two circuits shown in figure (A) and figure (B) below

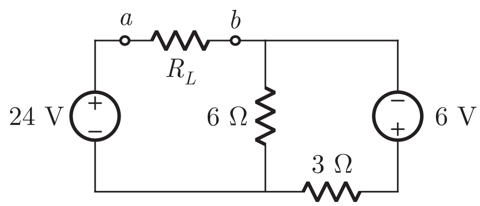


Fig.(A)

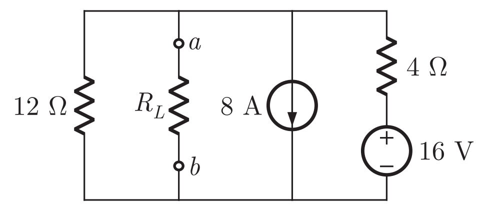


Fig.(B)

MCQ 5.1.28 The value of Thevenin voltage across terminals *a*-*b* of figure (A) and figure (B) respectively are

(A) 30 V, 36 V (B) 28 V, −12 V

(C) 18 V, 12 V (D) 30 V, −12 V

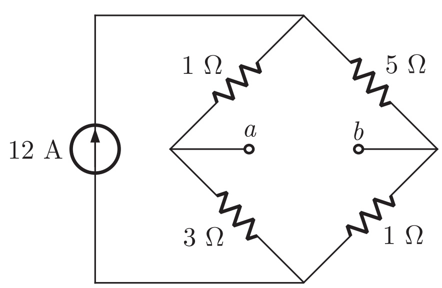
MCQ 5.1.29 The value of Thevenin resistance across terminals *a*-*b* of figure (A) and figure (B) respectively are

(A) zero, 3 Ω (B) 9 Ω, 16 Ω

(C) 2 Ω, 3 Ω (D) zero, 16 Ω

Statement for linked questions

Consider the circuit shown in the figure.



MCQ 5.1.30 The equivalent Thevenin voltage across terminal *a*-*b* is

(A) 31.2 V (B) 19.2 V

(C) 16.8 V (D) 24 V

MCQ 5.1.31 The Norton equivalent current with respect to terminal *a*-*b* is

(A) 13 A (B) 7 A

(C) 8 A (D) 10 A

MCQ 5.1.32 For a network having resistors and independent sources, it is desired to obtain Thevenin equivalent across the load which is in parallel with an ideal current source. Then which of the following statement is true ?

(A) The Thevenin equivalent circuit is simply that of a voltage source.

(B) The Thevenin equivalent circuit consists of a voltage source and a series resistor.

(C) The Thevenin equivalent circuit does not exist but the Norton equivalent does exist.

(D) None of these

MCQ 5.1.33 The Thevenin equivalent circuit of a network consists only of a resistor (Thevenin voltage is zero). Then which of the following elements might be contained in the network ?

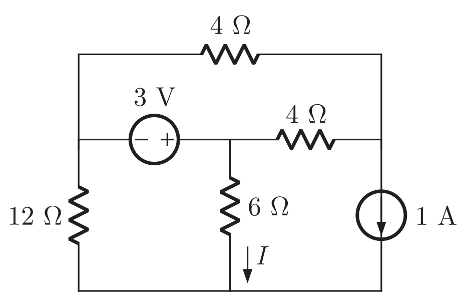
(A) resistor and independent sources

(B) resistor only

(C) resistor and dependent sources

(D) resistor, independent sources and dependent sources.

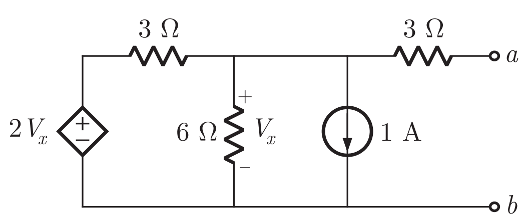
MCQ 5.1.34 In the following network, value of current I through $6\,\Omega$ resistor is given by



(A) 0.83 A (B) 2 A

(C) 1 A (D) -0.5 A

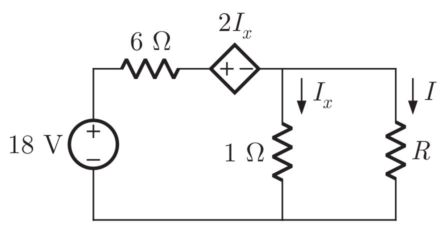
MCQ 5.1.35 For the circuit shown in the figure, the Thevenin's voltage and resistance looking into a-b are



(A) 2 V, $3\,\Omega$ (B) 2 V, $2\,\Omega$

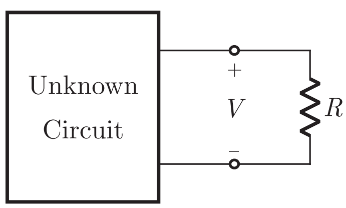
(C) 6 V, $-9\,\Omega$ (D) 6 V, $-3\,\Omega$

MCQ 5.1.36 For the circuit below, what value of R will cause $I = 2$ A ?



(A) $2/3\,\Omega$
(B) $4\,\Omega$
(C) zero
(D) none of these

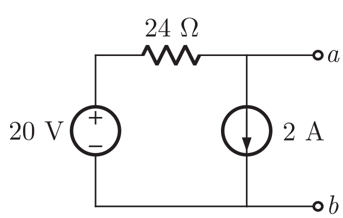
MCQ 5.1.37 For the following circuit, values of voltage V for different values of R are given in the table.



R	V
3 Ω	6 V
8 Ω	8 V

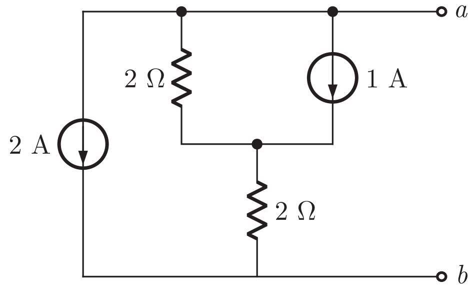
The Thevenin voltage and resistance of the unknown circuit are respectively.
(A) 14 V, $4\,\Omega$
(B) 4 V, $1\,\Omega$
(C) 14 V, $6\,\Omega$
(D) 10 V, $2\,\Omega$

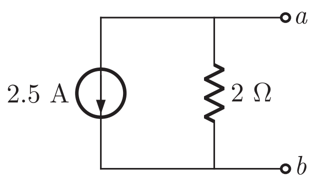
MCQ 5.1.38 In the circuit shown below, the Norton equivalent current and resistance with respect to terminal a-b is

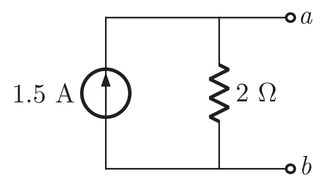


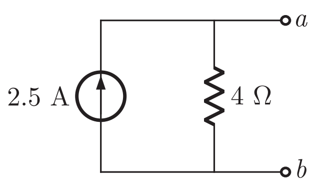
(A) $\frac{17}{6}$ A, $0\,\Omega$
(B) 2 A, $24\,\Omega$
(C) $-\frac{7}{6}$ A, $24\,\Omega$
(D) -2 A, $24\,\Omega$

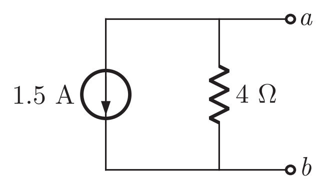
MCQ 5.1.39 The Norton equivalent circuit for the circuit shown in figure is given by



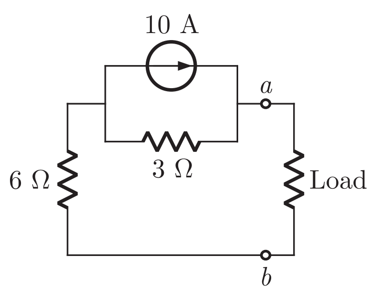
(A) 

(B) 

(C) 

(D) 

MCQ 5.1.40 What are the values of equivalent Norton current source (I_N) and equivalent resistance (R_N) across the load terminal of the circuit shown in figure ?



	I_N	R_N
(A)	10 A	2 Ω
(B)	10 A	9 Ω
(C)	3.33 A	9 Ω
(D)	6.66 A	2 Ω

MCQ 5.1.41 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in series parallel with an ideal voltage sources.

Consider the following statements :

1. Thevenin equivalent circuit across this terminal does not exist.
2. The Thevenin equivalent circuit exists and it is simply that of a voltage source.
3. The Norton equivalent circuit for this terminal does not exist.

Which of the above statements is/are true ?

(A) 1 and 3 (B) 1 only

(C) 2 and 3 (D) 3 only

MCQ 5.1.42 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in series with an ideal current sources.

Consider the following statements

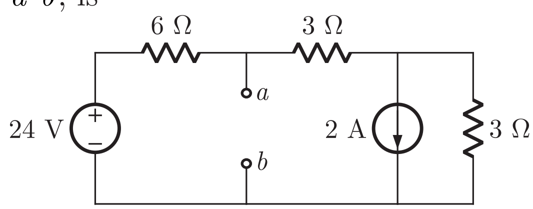
1. Norton equivalent across this terminal is not feasible.
2. Norton equivalent circuit exists and it is simply that of a current source only.
3. Thevenin's equivalent circuit across this terminal is not feasible.

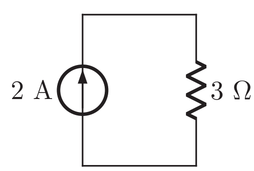
Which of the above statements is/are correct ?

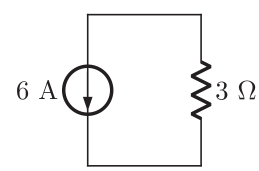
(A) 1 and 3 (B) 2 and 3

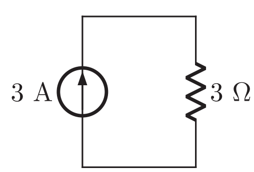
(C) 1 only (D) 3 only

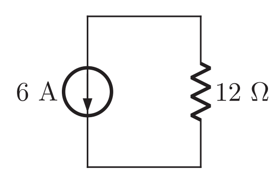
MCQ 5.1.43 The Norton equivalent circuit of the given network with respect to the terminal a-b, is



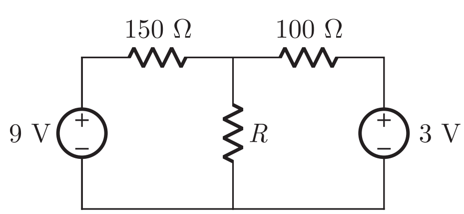
(A) 

(B) 

(C) 

(D) 

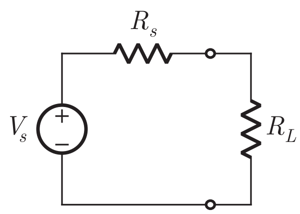
MCQ 5.1.44 The maximum power that can be transferred to the resistance R in the circuit is



(A) 486 mW (B) 243 mW

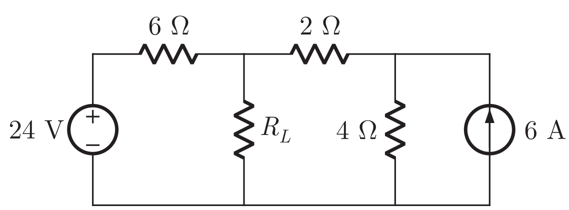
(C) 121.5 mW (D) 225 mW

MCQ 5.1.45 In the circuit below, if R_L is fixed and R_s is variable then for what value of R_s power dissipated in R_L will be maximum ?



(A) $R_S = R_L$
(B) $R_S = 0$
(C) $R_S = R_L/2$
(D) $R_S = 2R_L$

MCQ 5.1.46 In the circuit shown below the maximum power transferred to R_L is P_{max}, then



(A) $R_L = 12\,\Omega,\ P_{max} = 12\text{ W}$
(B) $R_L = 3\,\Omega,\ P_{max} = 96\text{ W}$
(C) $R_L = 3\,\Omega,\ P_{max} = 48\text{ W}$
(D) $R_L = 12\,\Omega,\ P_{max} = 24\text{ W}$

MCQ 5.1.47 In the circuit shown in figure (A) if current $I_1 = 2\text{ A}$, then current I_2 and I_3 in figure (B) and figure (C) respectively are

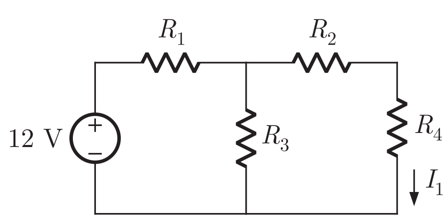


Fig.(A)

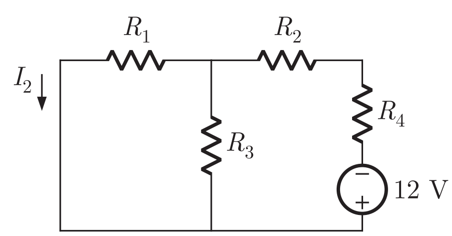


Fig.(B)

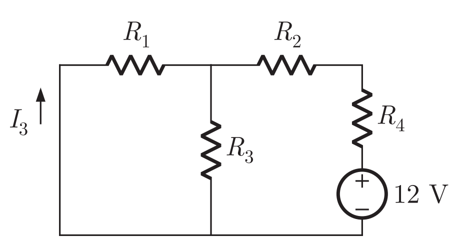


Fig.(C)

(A) 2 A, 2 A
(B) -2 A, 2 A
(C) 2 A, -2 A
(D) -2 A, -2 A

MCQ 5.1.48 In the circuit of figure (A), if $I_1 = 20$ mA, then what is the value of current I in the circuit of figure (B) ?

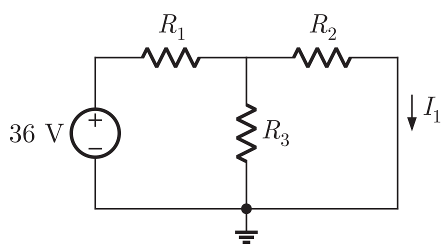


Fig.(A)

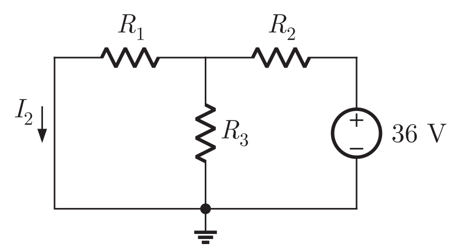


Fig.(B)

(A) 40 mA
(B) -20 mA
(C) 20 mA
(D) R_1, R_2 and R_3 must be known

MCQ 5.1.49 If $V_1 = 2$ V in the circuit of figure (A), then what is the value of V_2 in the circuit of figure (B) ?

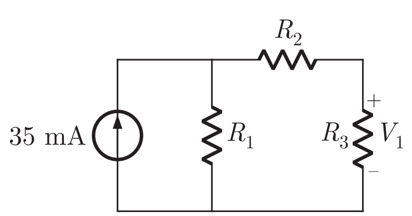


Fig.(A)

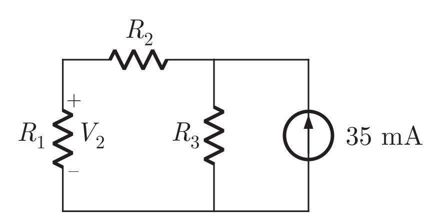
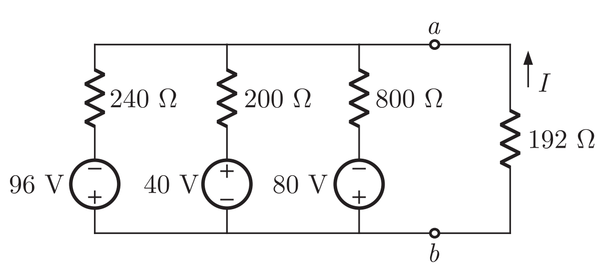


Fig.(B)

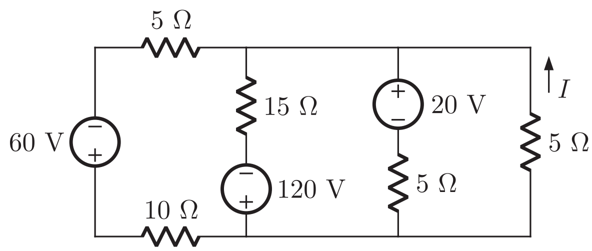
(A) 2 V
(B) -2 V
(C) 4 V
(D) R_1, R_2 and R_3 must be known

MCQ 5.1.50 The value of current I in the circuit below is equal to



(A) 100 mA
(B) 10 mA
(C) 233.34 mA
(D) none of these

MCQ 5.1.51 The value of current I in the following circuit is equal to



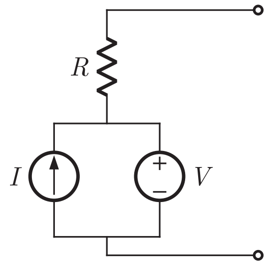
(A) 1 A (B) 6 A

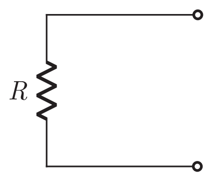
(C) 3 A (D) 2 A

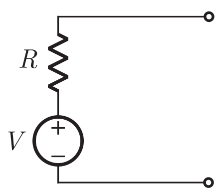
****

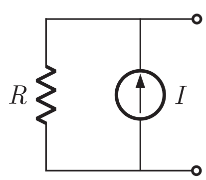
PRACTICE B

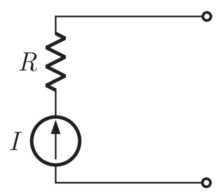
MCQ 5.2.1 A simple equivalent circuit of the two-terminal network shown in figure is



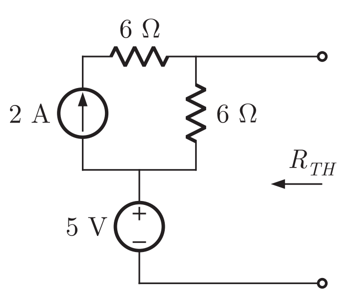
(A) 

(B) 

(C) 

(D) 

MCQ 5.2.2 For the following circuit the value of R_{Th} is



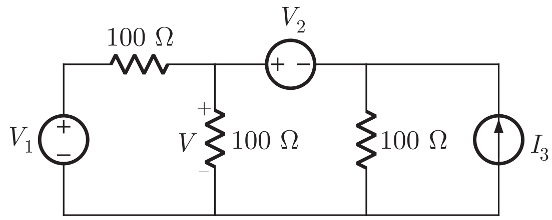
(A) 3 Ω

(B) 12 Ω

(C) 6 Ω

(D) ∞

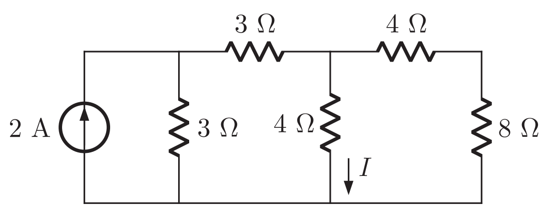
MCQ 5.2.3 If $V = AV_1 + BV_2 + CI_3$ in the following circuit, then values of A, B and C respectively are



(A) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ (B) $\frac{1}{3}, \frac{1}{3}, \frac{100}{3}$

(C) $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$ (D) $\frac{1}{3}, \frac{2}{3}, \frac{100}{3}$

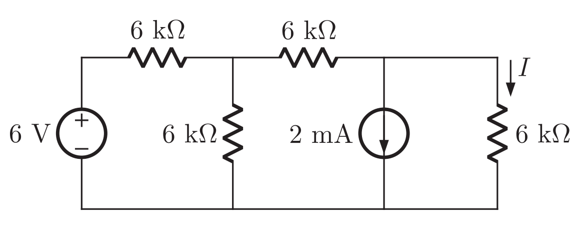
MCQ 5.2.4 What is the value of current I in the network of figure ?



(A) 0.67 A (B) 2 A

(C) 1.34 A (D) 0.5 A

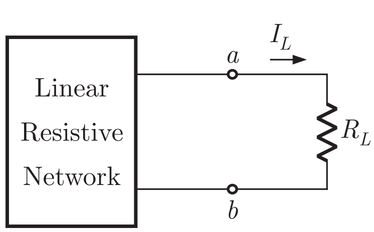
MCQ 5.2.5 The value of current I in the figure is



(A) -1 mA (B) 1.4 mA

(C) 1.8 mA (D) -1.2 mA

MCQ 5.2.6 For the circuit of figure, some measurements were made at the terminals a-b and given in the table below.



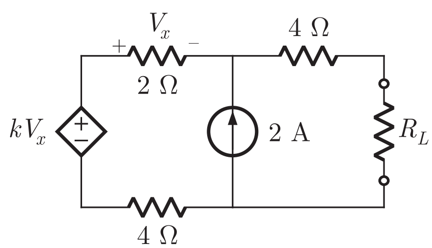
R_L	V_L
2 Ω	10 V
10 Ω	6 V

What is the value of I_L for $R_L = 20\,\Omega$?

(A) 3 A (B) 12 A

(C) 2 A (D) 4 A

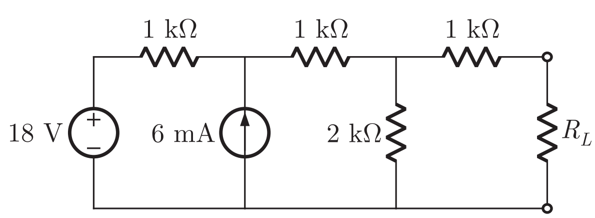
MCQ 5.2.7 In the circuit below, for what value of k, load $R_L = 2\,\Omega$ absorbs maximum power ?



(A) 4 (B) 7

(C) 2 (D) 6

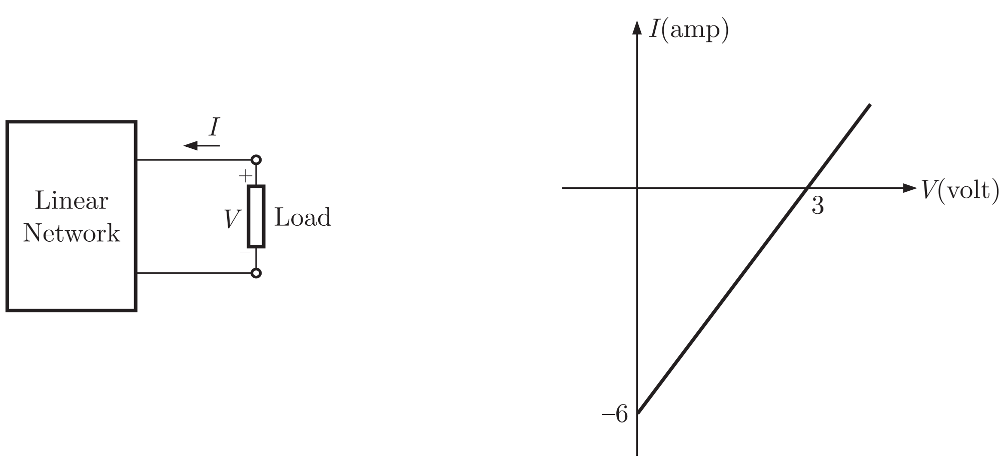
MCQ 5.2.8 In the circuit shown below, the maximum power that can be delivered to the load R_L is equal to



(A) 72 mW (B) 36 mW

(C) 24 mW (D) 18 mW

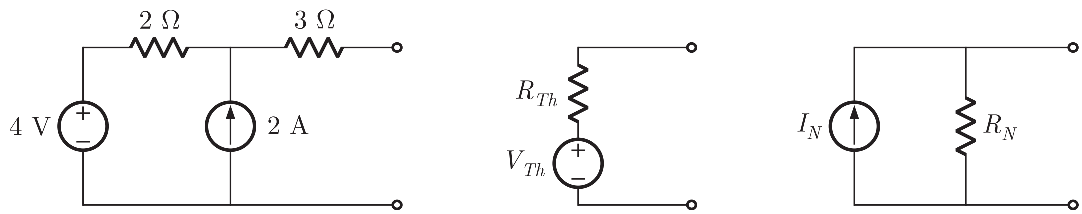
MCQ 5.2.9 For the linear network shown below, V-I characteristic is also given in the figure. The value of Norton equivalent current and resistance respectively are



(A) 3 A, 2 Ω (B) 6 Ω, 2 Ω

(C) 6 A, 0.5 Ω (D) 3 A, 0.5 Ω

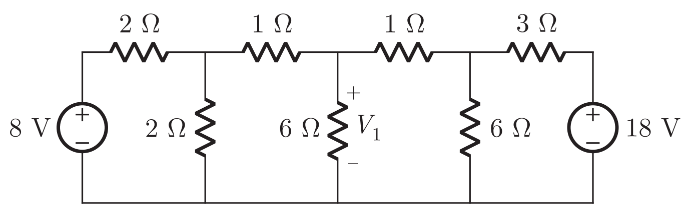
MCQ 5.2.10 In the following circuit a network and its Thevenin and Norton equivalent are given.



The value of the parameter are

	V_{Th}	R_{Th}	I_N	R_N
(A)	4 V	2 Ω	2 A	2 Ω
(B)	4 V	2 Ω	2 A	3 Ω
(C)	8 V	1.2 Ω	$\frac{30}{3}$ A	1.2 Ω
(D)	8 V	5 Ω	$\frac{8}{5}$ A	5 Ω

MCQ 5.2.11 In the following circuit the value of voltage V_1 is



(A) 6 V (B) 7 V

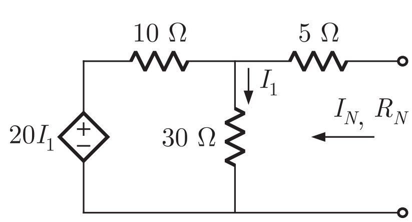
(C) 8 V (D) 10 V

MCQ 5.2.12 A practical DC current source provide 20 kW to a 50 Ω load and 20 kW to a 200 Ω load. The maximum power, that can drawn from it, is

(A) 22.5 kW (B) 45 kW

(C) 30.3 kW (D) 40 kW

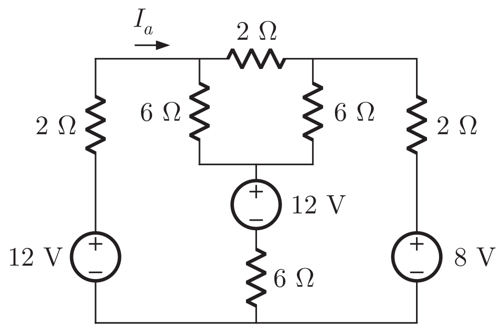
MCQ 5.2.13 For the following circuit the value of equivalent Norton current I_N and resistance R_N are

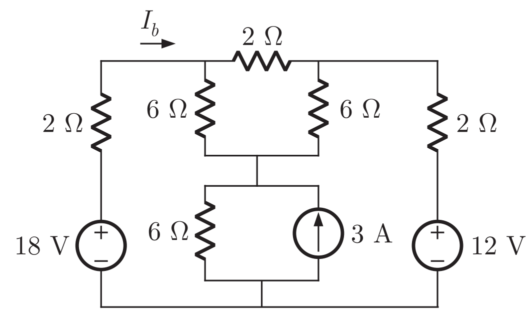


(A) 2 A, 20 Ω (B) 2 A, −20 Ω

(C) 0 A, 20 Ω (D) 0 A, −20 Ω

MCQ 5.2.14 Consider the following circuits shown below



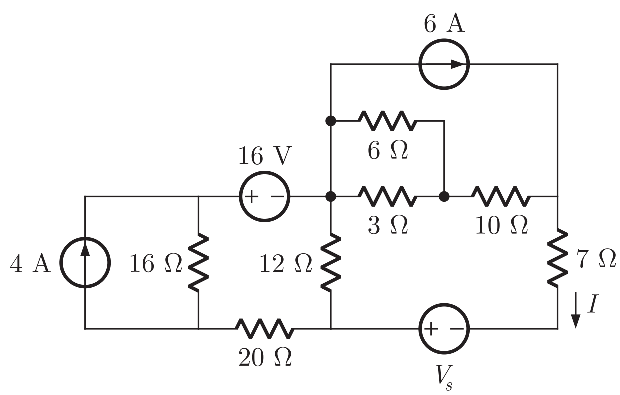


The relation between I_a and I_b is

(A) $I_b = I_a + 6$ (B) $I_b = I_a + 2$

(C) $I_b = 1.5 I_a$ (D) $I_b = I_a$

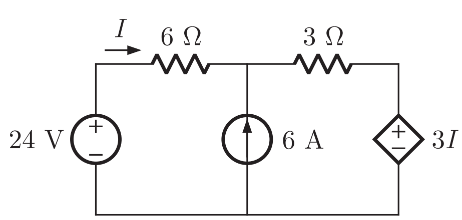
MCQ 5.2.15 If $I = 5$ A in the circuit below, then what is the value of voltage source V_s ?



(A) 28 V (B) 56 V

(C) 200 V (D) 224 V

MCQ 5.2.16 For the following circuit, value of current I is given by

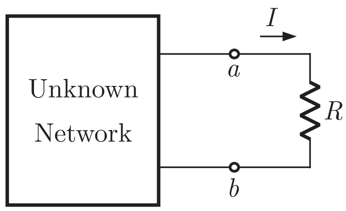


(A) 0.5 A (B) 3.5 A

(C) 1 A (D) 2 A

Statement for Linked Questions

In the following circuit, some measurements were made at the terminals a, b and given in the table below.



R	I
3 Ω	2 A
5 Ω	1.6 A

MCQ 5.2.17 The Thevenin equivalent of the unknown network across terminal a-b is

(A) 3 Ω, 14 V (B) 5 Ω, 16 V

(C) 16 Ω, 38 V (D) 10 Ω, 26 V

MCQ 5.2.18 The value of R that will cause I to be 1 A, is

(A) 22 Ω (B) 16 Ω

(C) 8 Ω (D) 11 Ω

MCQ 5.2.19 In the circuit shown in fig (a) if current $I_1 = 2.5$ A then current I_2 and I_3 in fig (B) and (C) respectively are

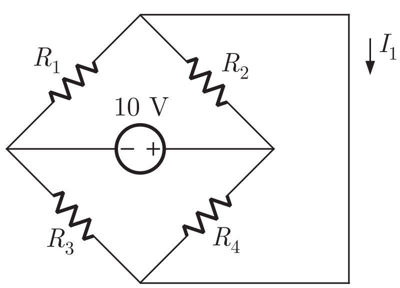


Fig.(A)

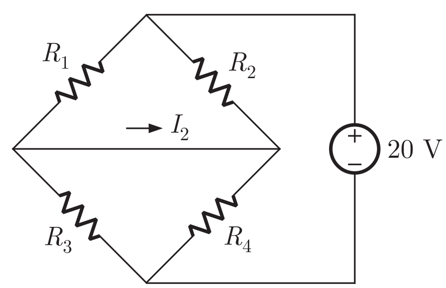


Fig.(B)

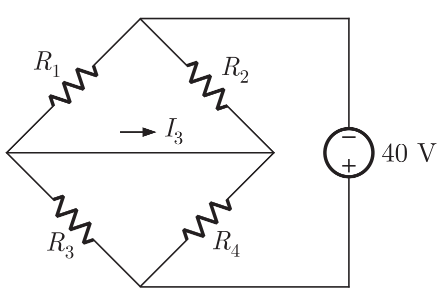
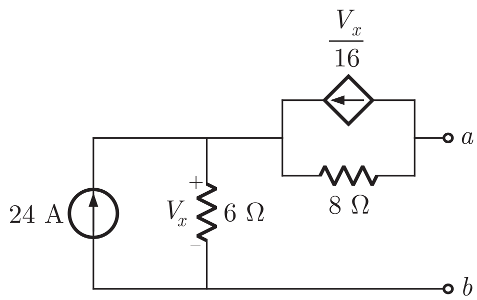


Fig.(C)

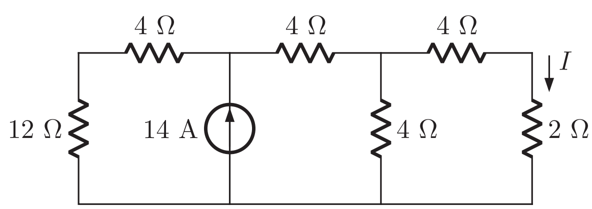
(A) 5 A, 10 A (B) −5 A, 10 A
(C) 5 A, −10 A (D) −5 A, −10 A

MCQ 5.2.20 The Thevenin equivalent resistance between terminal a and b in the following circuit is



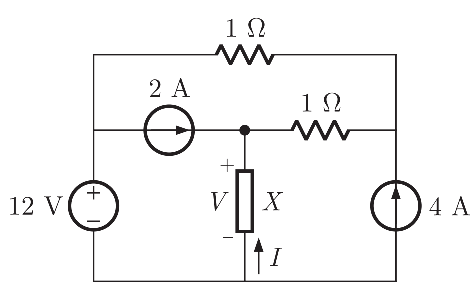
(A) 22 Ω (B) 11 Ω
(C) 17 Ω (D) 1 Ω

MCQ 5.2.21 In the circuit shown below, the value of current I will be given by



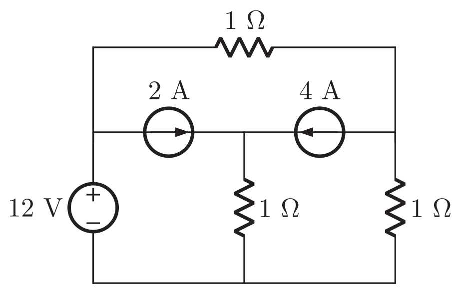
(A) 2.5 A (B) 1.5 A
(C) 4 A (D) 2 A

MCQ 5.2.22 The V-I relation of the unknown element X in the given network is $V = AI + B$. The value of A (in ohm) and B (in volt) respectively are



(A) 2, 20 (B) 2, 8
(C) 0.5, 4 (D) 0.5, 16

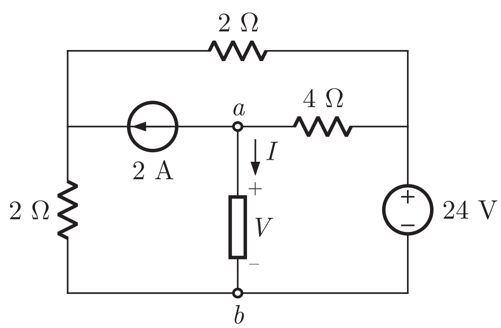
MCQ 5.2.23 The power delivered by 12 V source in the following network is



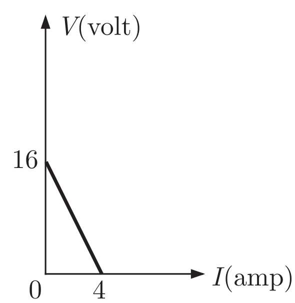
(A) 24 W (B) 96 W

(C) 120 W (D) 48 W

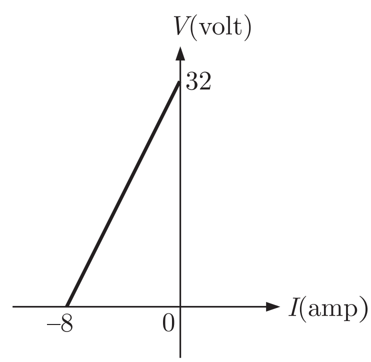
MCQ 5.2.24 For the following network the V-I curve with respect to terminals a-b, is given by



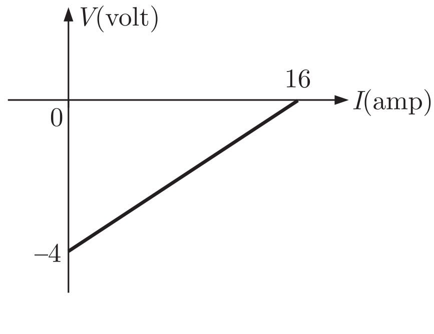
(A)



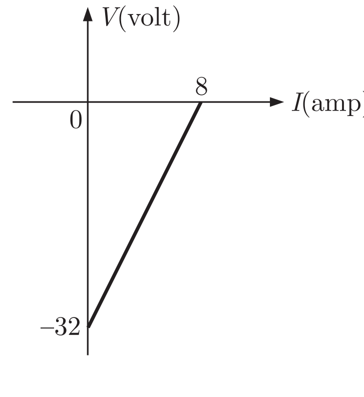
(B)



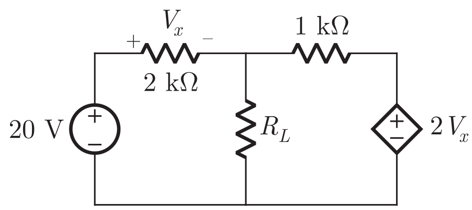
(C)



(D)



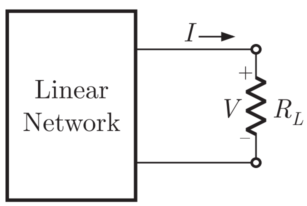
MCQ 5.2.25 In the circuit shown, what value of R_L maximizes the power delivered to R_L ?

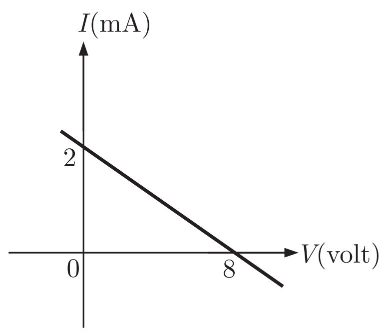


(A) 286 Ω (B) 350 Ω

(C) zero (D) 500 Ω

MCQ 5.2.26 The V-I relation for the circuit below is plotted in the figure. The maximum power that can be transferred to the load R_L will be

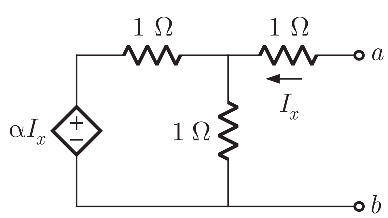




(A) 4 mW (B) 8 mW

(C) 2 mW (D) 16 mW

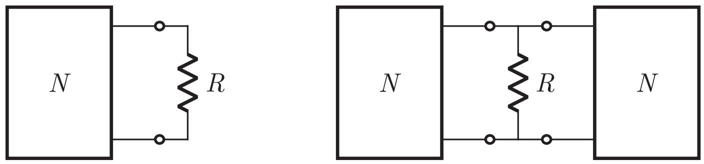
MCQ 5.2.27 In the following circuit equivalent Thevenin resistance between nodes a and b is $R_{Th} = 3\,\Omega$. The value of α is



(A) 2 (B) 1

(C) 3 (D) 4

MCQ 5.2.28 A network N feeds a resistance R as shown in circuit below. Let the power consumed by R be P. If an identical network is added as shown in figure, the power consumed by R will be



(A) equal to P
(B) less than P
(C) between P and $4P$
(D) more than $4P$

MCQ 5.2.29 A certain network consists of a large number of ideal linear resistors, one of which is R and two constant ideal source. The power consumed by R is P_1 when only the first source is active, and P_2 when only the second source is active. If both sources are active simultaneously, then the power consumed by R is

(A) $P_1 \pm P_2$
(B) $\sqrt{P_1} \pm \sqrt{P_2}$
(C) $(\sqrt{P_1} \pm \sqrt{P_2})^2$
(D) $(P_1 \pm P_2)^2$

MCQ 5.2.30 If the $60\,\Omega$ resistance in the circuit of figure (A) is to be replaced with a current source I_s and $240\,\Omega$ shunt resistor as shown in figure (B), then magnitude and direction of required current source would be

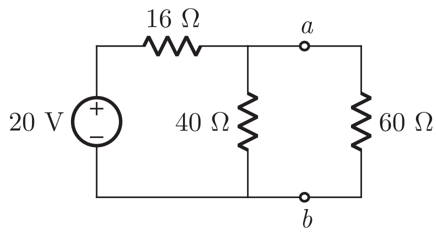


Fig.(A)

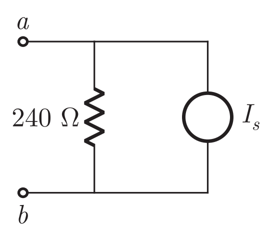
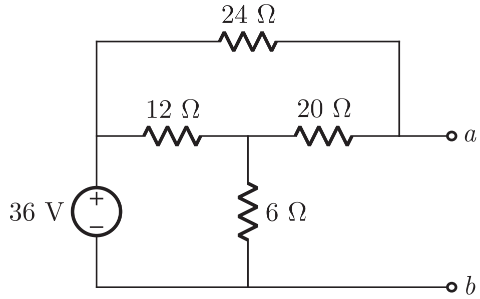


Fig.(B)

(A) 200 mA, upward
(B) 150 mA, downward
(C) 50 mA, downward
(D) 150 mA, upward

MCQ 5.2.31 The Thevenin's equivalent of the circuit shown in the figure is

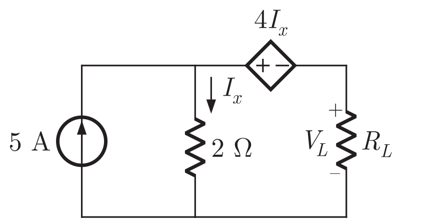


(A) $4\,\text{V}$, $48\,\Omega$
(B) $24\,\text{V}$, $12\,\Omega$
(C) $24\,\text{V}$, $24\,\Omega$
(D) $12\,\text{V}$, $12\,\Omega$

MCQ 5.2.32 The voltage V_L across the load resistance in the figure is given by

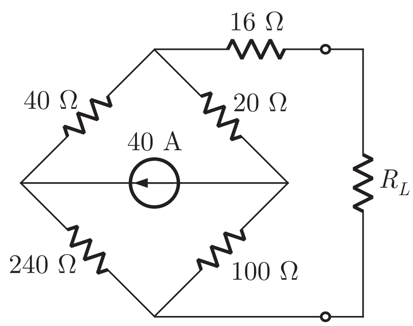
$$V_L = V\left(\frac{R_L}{R+R_L}\right)$$

V and R will be equal to



(A) -10 V, $2\,\Omega$
(B) 10 V, $2\,\Omega$
(C) -10 V, $-2\,\Omega$
(D) none of these

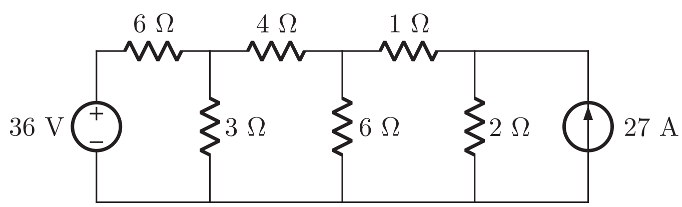
MCQ 5.2.33 The maximum power that can be transferred to the load resistor R_L from the current source in the figure is



(A) 4 W
(B) 8 W
(C) 16 W
(D) 2 W

Common data for Q. 34 to Q. 35

An electric circuit is fed by two independent sources as shown in figure.



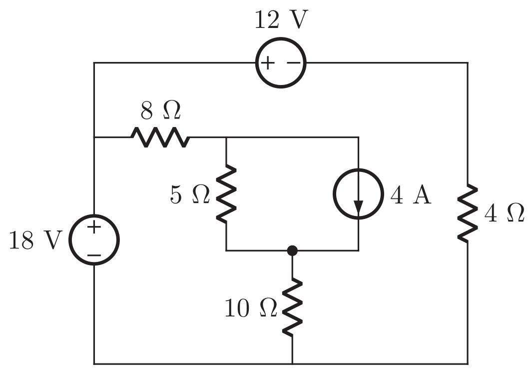
MCQ 5.2.34 The power supplied by 36 V source will be
(A) 108 W
(B) 162 W
(C) 129.6 W
(D) 216 W

MCQ 5.2.35 The power supplied by 27 A source will be

(A) 972 W (B) 1083 W

(C) 1458 W (D) 1026 W

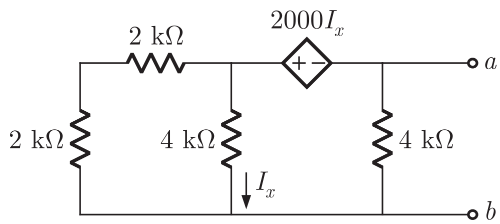
MCQ 5.2.36 In the circuit shown in the given figure, power dissipated in $4\,\Omega$ resistor is



(A) 225 W (B) 121 W

(C) 9 W (D) none of these

MCQ 5.2.37 In the circuit given below, viewed from a-b, the circuit can be reduced to an equivalent circuit as



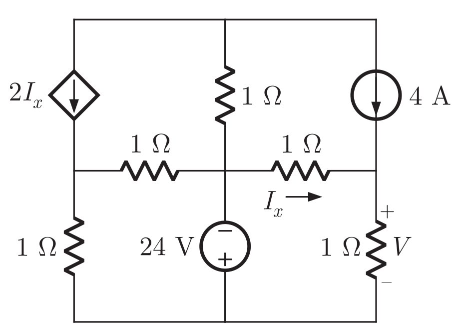
(A) 10 volt source in series with $2\,k\Omega$ resistor

(B) $1250\,\Omega$ resistor only

(C) 20 V source in series with $1333.34\,\Omega$ resistor

(D) $800\,\Omega$ resistor only

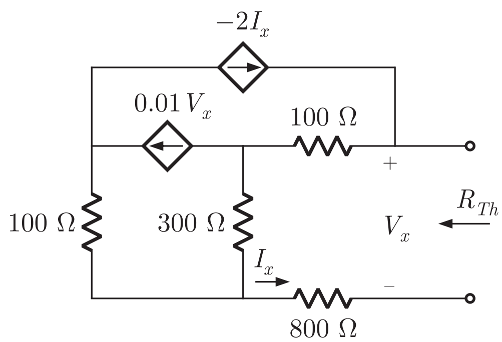
MCQ 5.2.38 What is the value of voltage V in the following network ?



(A) 14 V (B) 28 V

(C) -10 V (D) none of these

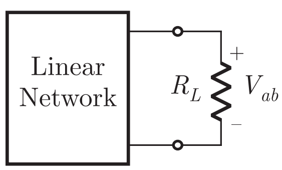
MCQ 5.2.39 For the circuit shown in figure below the value of R_{Th} is



(A) 100 Ω (B) 136.4 Ω

(C) 200 Ω (D) 272.8 Ω

MCQ 5.2.40 Consider the network shown below :



The power absorbed by load resistance R_L is shown in table :

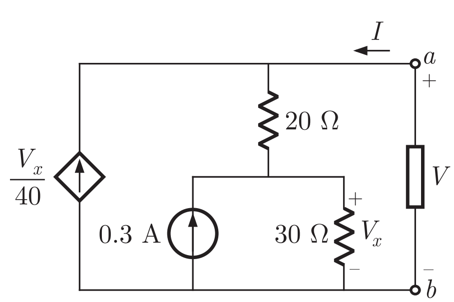
R_L	10 kΩ	30 kΩ
P	3.6 MW	1.5 A

The value of R_L, that would absorb maximum power, is

(A) 60 kΩ (B) 100 Ω

(C) 300 Ω (D) 30 kΩ

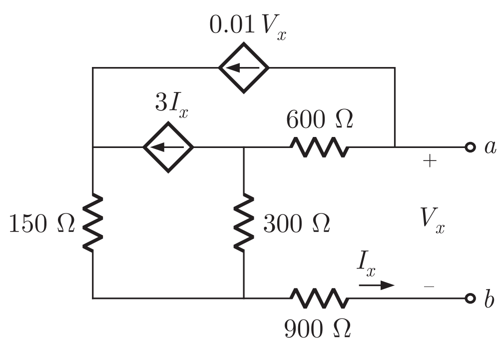
MCQ 5.2.41 The V-I equation for the network shown in figure, is given by



(A) $7V = 200I + 54$ (B) $V = 100I + 36$

(C) $V = 200I + 54$ (D) $V = 50I + 54$

MCQ 5.2.42 In the following circuit the value of open circuit voltage and Thevenin resistance at terminals a, b are



(A) $V_{OC} = 100$ V, $R_{Th} = 1800\ \Omega$ (B) $V_{OC} = 0$ V, $R_{Th} = 270\ \Omega$

(C) $V_{OC} = 100$ V, $R_{Th} = 90\ \Omega$ (D) $V_{OC} = 0$ V, $R_{Th} = 90\ \Omega$

****